HOUSING PRICE CHANGES, GENERAL EQUILIBRIUM AND WELFARE

Ashot Tsharakyan – Martin Janičko

Abstract

This paper explores the aggregate welfare effects of housing price changes in the heterogeneous agent general equilibrium model with multi-sector production side. The model includes two types of households: credit-constrained households and unconstrained households. These types differ not only because of the presence or absence of credit constraints but also from the point of view of their time preference rates and factors of production which they own. The modeling of the production side of the economy is based on Davis and Heathcote (2005) and includes a composite good production sector housing production sector and intermediate goods production sector. Besides welfare comparisons between steady states, the welfare changes during transition between steady states are also calculated.

KEY WORDS: general equilibrium, housing price changes, aggregate welfare, binding credit constraints, multi-sector production side, construction regulations

JEL CLASSIFICATION: C68, R20, R31, R28, G15

Introduction

Over the last 14 years, the US housing market has been characterized by drastic changes in housing prices. In particular, in the period from 1995 to 2006, according to National Association of Realtors, the median house price increased by unprecedented 190%, i.e. almost tripling. However, starting from 2007, because of the financial crisis and bust in the housing market, the trend has reversed and the median house price has decreased by around 52%.

These enormous housing price shocks have had considerable implications not only for the financial stability of the country and of the global economy, but also for household consumption and welfare, which were explored in the previous literature both for individual groups of households as well as on the aggregate level. For exploring the effects of housing price changes on consumption and welfare of separate groups of households, mainly life-cycle models of housing choice have been used. For instance, Campbell and Cocco (2005), based on the life-cycle model and UK micro-level data on real non-durable consumption growth and real housing price growth, demonstrate high positive correlation between an increase in the growth rate of housing prices and growth rate of non-durable consumption. Furthermore, Li and Yao (2004) also employ a life-cycle model of housing tenure choice, and find that for homeowners less than 40 years old, an increase in housing prices leads to welfare losses, while in case of older homeowners it leads to an increase in both their welfare as well as consumption. Equally, Kiyotaki and Michaelides (2007) develop an open-economy life-cycle model of a production economy where residential and commercial...
structures are built by using land and capital. Using the model find that a permanent increase in the growth rate of labor productivity and a decrease in the world real interest rates substantially redistribute wealth from net buyers of houses to net sellers with a housing price hike. Bajari et al. (2005) explore the aggregate welfare effects of housing price appreciation. In this paper authors measure changes in consumer welfare due to changes in the prices of owner-occupied housing by means of income required to keep expected discounted utility constant. They also consider only exogenous changes in housing prices and assume that households are not credit-constrained. The authors show that there is no change in aggregate welfare due to an increase in the price of the existing stock of housing. Finally, Tsharakyan and Janíčko (2010) also analyze the effects of housing price appreciation on aggregate welfare, but generalize the previously available results by incorporating credit constraints and endogenous housing price into welfare effects calculation. At first the credit constraint is incorporated into the model with endogenous housing price, and it is shown that in this model housing price appreciation leads to an improvement in aggregate welfare due to the effect of credit constraint relaxation resulting from housing price appreciation. Then the housing price is endogenized by modeling the supply side of the housing market. Finally the demand and supply shocks causing housing price appreciation are calibrated according to US housing market data from years 1995-2006, and it is demonstrated that housing price appreciation is driven by the given combination of demand and supply shocks still leads to improvement in aggregate welfare.

It is crucial to note that while Tsharakyan and Janíčko (2010) keep the income formation exogenous, do not model the composite good production sector and use Bajari’s definition of welfare adjustment, the present paper analyzes the aggregate welfare effects of housing price changes in a full general equilibrium environment. It contributes to the previous literature by building a heterogeneous agent general equilibrium model in which the aggregate welfare effects of housing price changes can be studied in a more comprehensive way. The model includes two types of households: the credit-constrained ones and the unconstrained ones. These types differ not only because of the presence or absence of credit constraints, but also from the point of view of their time preference rates and factors of production which they own. Incorporation of differential time preference rates (and consequently discount factors) is based on Kiyotaki and Moore (1997) and insures that in equilibrium more patient unconstrained households will lend their extra funds to credit-constrained households and the credit market will clear, assuming the economy with a unique equilibrium. All the factors of production, i.e. capital, land and labor, are owned by households and are supplied to the firms for production. There are two goods in this economy: housing and composite consumption good. Modeling of the production side of the economy is based on Davis and Heathcote (2005) and includes the composite good production sector, the housing production sector, and the intermediate good production sector.

After the model is defined, the steady state is calculated. Then it is explored what happens with aggregate welfare when different demand and supply-side shocks cause changes in housing price and economy transfers to a new steady state. Sources of housing price shocks include changes in productivity of different production sectors and changes in maximum loan-to-value ratios. Change in aggregate welfare during transition and also change in aggregate welfare in the new steady state compared with the old steady state are calculated. Finally, both the effects of housing price appreciation as well as the effects of housing price decline, which is currently characteristic for the US housing market, are considered and their impact is properly elaborated on.
The rest of the article is organized as follows. Section 2 describes the model and defines equilibrium. Section 3 contains the calibration. Section 4 reports the numerical results.

2. The Model

2.1 Production sector

Modeling of the production of housing and composite good is based on Davies and Heathcote (2005), but is simplified for the purposes of the present paper. Perfectly competitive intermediate goods producing firms use capital rented from the household and labor supplied by the households to produce intermediate goods: construction (denoted by c), manufacturing (denoted by m) and services (denoted by s). The intermediate goods are produced using standard Cobb-Douglas technology. The production of manufacturing good is subject to a productivity shock denoted by \( z_t \). The productivity shock follows standard AR(1) process, which is calibrated later. The production function for manufacturing good is given by

\[
Y_m = z_t K_m^\alpha N_m^{1-\alpha} + \xi_t,
\]

where \( K \) stands for capital and \( N \) stands for labor. The maximization problem for manufacturing good producers is then given by

\[
\max_{\{K_m, N_m\}} \left\{ p_m z_t K_m^\alpha N_m^{1-\alpha} - w_t N_m - r_t K_m \right\},
\]

s.t.

\[
K_m, N_m \geq 0
\]

and

\[
z_t = z + az_{t-1} + \xi_t,
\]

The profit maximizing conditions for manufacturing good producing firms are given by

\[
p_m \alpha_m z_t K_m^\alpha N_m^{1-\alpha} = r_t,
\]

\[
p_m (1-\alpha_m) z_t K_m^\alpha N_m^{\alpha} = w_t.
\]

The production function for the remaining intermediate goods is given by \( Y_j = K_j^\alpha N_j^{1-\alpha} \), where \( K \) stands for capital and \( N \) stands for labor. The maximization problem for manufacturing good producers is then given by

\[
\max_{\{K_j, N_j\}} \left\{ p_j K_j^\alpha N_j^{1-\alpha} - w_j N_j - r_j K_j \right\}, \text{ for } j \in \{c, s\}
\]

s.t.

\[
K_j, N_j \geq 0
\]
The profit maximizing conditions for manufacturing good producing firms are given by

\begin{equation}
 p_{j,t} \alpha_j K_{j,t}^{\alpha_j - 1} N_{j,t}^{1-\alpha_j} = r_t,
\end{equation}

\begin{equation}
 p_{j,t} (1-\alpha_j) K_{j,t}^{\alpha_j - 1} N_{j,t}^{1-\alpha_j} = w_t,
\end{equation}

The goods produced by intermediate good producers are used as inputs by final good producers for production of a composite consumption good and a residential investment good. Let us denote by subscript \( co \) the consumption good and by subscript \( res \) the residential investment good. The production function for final good \( f \) is given by

\begin{equation}
 X_{sf,t}^{\alpha_f} X_{mf,t}^{\beta_f} X_{ms,t}^{1-\alpha_f - \beta_f},
\end{equation}

where \( X_{sf,t}, X_{mf,t} \) and \( X_{sf,t} \) denote quantity of, correspondingly, construction, manufacturing and services used in the production of the final good \( f \). The final good producer's problem is given by:

\begin{equation}
 \max \{ p_{f,t} X_{sf,t}^{\alpha_f} X_{mf,t}^{\beta_f} X_{ms,t}^{1-\alpha_f - \beta_f} - w_t N_{j,t} - r_t K_{j,t} \},
\end{equation}

s.t.

\begin{equation}
 X_{sf,t}, X_{mf,t}, X_{sf,t} \geq 0
\end{equation}

F.O.C. for this problem are given by

\begin{equation}
 p_{f,t} \alpha_f X_{sf,t}^{\alpha_f - 1} X_{mf,t}^{\beta_f} X_{ms,t}^{1-\alpha_f - \beta_f} = p_{c,t}
\end{equation}

\begin{equation}
 p_{f,t} \beta_f X_{sf,t}^{\alpha_f} X_{mf,t}^{\beta_f - 1} X_{ms,t}^{1-\alpha_f - \beta_f} = p_{m,t}
\end{equation}

\begin{equation}
 p_{f,t} (1-\alpha_f - \beta_f) X_{sf,t}^{\alpha_f} X_{mf,t}^{\beta_f} X_{ms,t}^{1-\alpha_f - \beta_f} = p_{s,t}
\end{equation}

The housing construction sector combines residential investment good with land to produce housing units. It is subject to sector-specific productivity shock. The introduction of a specific productivity shock is intended for generating negative supply shock in the housing production. Being more specific, according to Glaeser and Guyourko (2005), this was characteristic for the US of 1990s and played an important role in the observed housing price dynamics. Essentially, they argue that in 1990s new housing construction in the US was considerably limited by increasing difficulty of obtaining regulatory approval for building houses. This can be attributed to changing judicial tastes (that is willingness of judicial
authorities to reject building permit approvals), increasing political pressures of existing homeowners, decreasing ability to bribe regulators, and rising environmental concerns. Such changes made the process of getting building permit for developers more costly both in monetary terms as well as in terms of time, or in other words, increased implicit costs of housing construction. Therefore, in our paper, the increase in the strictness of building permit regulation works through decreasing productivity in housing production sector. Moreover, following Saiz (2010), the level of strictness of regulatory restrictions is determined endogenously depending on the housing price level and the net change in housing demand that is investment of households into new housing. Such determination of the degree of regulation tightness is quite logical since in case of higher demand pressure or lower price of the housing the political pressure of existing homeowners against new construction as well as environmental concerns and other factors should be stronger. Denoting regulation variable by $rg$, we assume that regulation strictness level is determined according to $rg_t = \psi q_{t-1} + \chi x_{t-1}$, where $x_{t-1} = x_{c,t-1} + g x_{u,t-1}$, while $\psi$ and $\chi$ are constants calibrated later. When determining the process for productivity per se, it is assumed that it could change not only because of regulation but also because of production specific factors. Thus in my model productivity in housing sector in period $t$ denoted by $\eta_t$ is dependent both on regulation strictness $rg$ as well as on its previous period value. That is, equation for productivity in housing sector is given by

$$ Y_{ht} = \eta_t (X_{res,t})^\varepsilon (La_t)^{1-\varepsilon}, \quad \text{where} \quad X_{res,t} \quad \text{stands for the amount of residential investment good used as input in production of housing units and} \quad La_t \quad \text{stands for the amount of land used. The profit maximization of construction firm is thus given by}.$$

$$ \max_{(X_{res,t}, La_t)} \left[ q_t \eta_t (X_{res,t})^\varepsilon (La_t)^{1-\varepsilon} - p_t X_{res,t} - p_{l,t} La_t \right], \quad (16) $$

$$ \text{s.t.} \quad X_{res,t}, La_t \geq 0, \quad (17) $$

$$ rg_t = \psi \log q_{t-1} + \chi \log x_{t-1}, \quad (68) $$

$$ \eta_t = \sigma + \rho \eta_{t-1} + \phi rg_t + \zeta_t, \quad (79) $$

where $q_t$ stands for the price of a housing unit and $p_{l,t}$ stands for the price of land. The profit maximizing conditions for housing construction firms are given by:

$$ q_t \eta_t X_{res,t}^{1-\varepsilon} La_t^{1-\varepsilon} = p_{res}; \quad (20) $$

$$ q_t \eta_t (1-\varepsilon) X_{res,t}^{\varepsilon} La_t^{-\varepsilon} = p_{l,t}. \quad (21) $$

2.2 Households
There are two types of households in the model, namely credit constrained households with a population of size 1 and unconstrained households with a population of size g. The most important difference between these types is, correspondingly, the presence and absence of credit constraints in their optimization problems. In addition to ensure that in equilibrium unconstrained households will lend funds to constrained ones, a different structure of owned factors of production and different rates of time preference for each of the types are assumed. Both credit constrained and unconstrained households own land and the total amount of land in the economy, \( \tilde{L} \), is evenly distributed between and among households. Constrained households supply labor to the intermediate good producing firms. Here, the inelastic labor supply case is considered and labor supply is normalized to 1. Constrained households derive utility from consumption of housing and the composite consumption good and their preferences are denoted by \( u(c_{c,t}, h_{c,t}) \). The composite consumption good is considered *numeraire* and its price is normalized to 1. Constrained households can invest into risk-free bonds and if the bond holdings chosen by them are negative, it means that households are borrowers. The discount factor of credit constrained households is denoted by \( \beta^c \).

Constrained households are subject to credit constraint of the form \( b_{c,t+1} \geq -mq_{c,t+1} \), implying that in each period households can borrow only a certain fraction \( m \) of the current value of their housing. When solving the model and simulating transitional dynamics, the credit constraint is used with strict equality. This means that in this paper, credit constrained households are those who have to borrow up to the maximum limit when financing a housing purchase. On one hand it can be interpreted as the upper limit of the degree of being credit constrained, but on the other hand it rules out the households who have enough cash to buy house without a mortgage but find it more profitable in terms of net present value to finance their housing purchase with a mortgage. Such households would typically not borrow the maximum possible amount since this implies a higher interest rate. Thus, only the households that have enough savings for a low down payment and have to borrow the rest are considered credit-constrained.

Housing depreciates at a constant rate \( \delta_h \). In what follows the allocations chosen by credit-constrained households are distinguished by subscript \( c \). Households choose how many bonds to carry into the next period, \( b_{c,t+1} \), how much housing to carry into next period \( h_{c,t+1} \), and how much to consume in current period, \( c_{c,t} \). Based on the assumptions above the constrained household problem can be formulated as follows:

\[
V_c(h_{c,t}, b_{c,t}, \eta_t, z_t) = \max_{\{c_{c,t}, h_{c,t+1}, b_{c,t+1}\}} \{u(c_{c,t}, h_{c,t}) +
+ \beta^c E_t V_c(h_{c,t+1}, b_{c,t+1}, \eta_{t+1}, z_{t+1})\},
\]

s.t.
\[
c_{c,t} + q x_{c,t} + s_{c,t} = w_t + p_{l,t} (\tilde{L}/(1+g)) + i b_{c,t},
\]
\[
b_{c,t+1} - b_{c,t} = s_{c,t},
\]
\[ h_{c,t+1} - h_{c,t} = x_{c,t} - \delta_t h_{c,t}, \]  
(25)

\[ b_{c,t+1} \geq -mq_t h_{c,t+1}. \]  
(26)

Taking FOCs, rearranging, and using utility function of the form 
\[ u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} + \theta \frac{h^{1-\gamma}}{1-\gamma}, \] 
based on Campbell and Cocco (2004) yields the following Euler equations for credit-constrained households:

\[ \nu_t = c_{c,t}^{-\gamma} - \beta \theta E_t c_{c,t+1}^{-\gamma} (1+i_{t+1}), \]  
(87)

\[ q_t c_{c,t}^{-\gamma} = \beta \nu c_{c,t+1} - \beta \theta E_t c_{c,t+1}^{-\gamma} q_{t+1} (1-\delta_h) + \]  
\[ + mq_t \nu_t, \]  
(98)

where \( \nu_t \) is the multiplier of credit constraint.

Each of unconstrained households possesses the same quantity of land as a constrained one. Each of them supplies one unit of labor to the intermediate good producers. In addition, unconstrained households own capital which they supply to the intermediate good producers. Assuming an additional source of income for the unconstrained households is on one hand justified from the modeling perspective, ensuring that they have additional wealth to lend in the equilibrium, and on the other hand by the fact that in real life, unconstrained households usually have higher net worth than constrained households. Capital depreciates at rate \( \delta_k \).

Investment of unconstrained households into capital is denoted by \( I_t \). The allocations made by unconstrained households are denoted by subscript \( u \). To ensure that unconstrained households have incentives to lend, it is assumed that unconstrained households have low impatience so their discount factor is higher than that of the constrained households. The discount factor of unconstrained households is denoted by \( \beta^u \). Unconstrained households choose how many bonds to carry into the next period, \( b_{u,t+1} \), how much housing to carry into next period, \( h_{u,t+1} \), how much to consume in current period, \( c_{u,t} \), and how much capital to carry into the next period, \( h_{u,t+1} \). The optimization problem of unconstrained households is given by:

\[ V_u(h_{u,t}, b_{u,t}, k_t, \eta_t, z_t) = \max_{c_{u,t}, h_{u,t+1}, b_{u,t+1}, k_{t+1}} \{ u(c_{u,t}, h_{u,t}) + \]  
\[ + \beta^u E_t V_u(h_{u,t+1}, b_{u,t+1}, k_{t+1}, \eta_{t+1}, z_{t+1}) \}, \]  
(109)

s.t.

\[ c_{u,t} + q_t x_{u,t} + s_{u,t} + I_t = w_t + p_{i,t} \bar{L}(1+g) + i b_{u,t} + r k_t, \]  
(30)

\[ b_{u,t+1} - b_{u,t} = s_{u,t}, \]  
(31)

\[ h_{u,t+1} - h_{u,t} = x_{u,t} - \delta_t h_{u,t}, \]  
(32)
\[ k_{t+1} - k_t = I_t - \delta_k k_t. \] (32)

Taking FOCs, rearranging, and using the utility function above yields the following Euler equations for unconstrained households:

\[ c^{\text{\#}}_{u,t} = \beta^\alpha E_t c^{\text{\#}}_{u,t+1} (1 + i_{t+1}), \] (31)

\[ q_t c^{\text{\#}}_{u,t} = \beta^\alpha \theta E_t h^{\text{\#}}_{u,t+1} + \beta^\alpha E_t c^{\text{\#}}_{u,t+1} q_{t+1} (1 - \delta_h), \] (32)

\[ c^{\text{\#}}_{u,t} = \beta^\alpha E_t c^{\text{\#}}_{u,t+1} (1 + r_{t+1} - \delta_r). \] (33)

2.3 Definition of equilibrium

The equilibrium consists of prices \( \{q_t, r_t, w_t, P_{s,t}, P_{c,t}, P_{m,t}, P_{res,t}, P_{l,t}\} \), shadow price of credit constraint \( \{u_t\} \), interest rate \( \{i_t\} \), allocations \( \{c_{c,t}, h_{c,t}, b_{c,t+1}, c_{a,t}, h_{a,t+1}, b_{a,t+1}, k_{t+1}\} \) by households and the profit maximizing input demands of firms \( \{K_{s,t}, K_{m,t}, K_{c,t}, N_{s,t}, N_{m,t}, N_{c,t}, La_t, X_{res,t}, X_{eco,t}, X_{mcot,t}, X_{sco,t}, X_{cre,t}, X_{sres,t}, X_{nres,t}\} \) and level of regulation \( \{r_{gt}\} \) such that

1) given prices, households solve their optimization problem (conditions (27)-(98) and (31)) and firms maximize their profits (conditions (Chyba! Nenalezen zdroj odkazů.) - (33)) and 2) Markets clear,

i) (housing market),

\[ x_{c,t} + g x_{u,t} = Y_{h,t} \] (11)

ii) (composite good market),

\[ c_{c,t} + g c_{u,t} + g I_t = Y_{co,t} \] (125)

iii) (capital market),

\[ K_{m,t} + K_{n,t} + K_{s,t} = g k_t \] (136)

iv) (credit market),

\[ b_{c,t+1} = -g b_{u,t+1} \] (147)

v) (labor market),

\[ N_{c,t} + N_{s,t} + N_{m,t} = g + 1 \] (158)
vi)  

\begin{align*}
X_{eco,t} + X_{cre,t} &= Y_{c,t} \\
X_{meo,t} + X_{mres,t} &= Y_{m,t} \\
X_{sco,t} + X_{sres,t} &= Y_{s,t}
\end{align*}  

(Intermediate goods market),

vii)  

\[ L a_t = \bar{L} \]  

(Land market).

vii)  

\[ X_{res,t} = Y_{res,t} \]  

(Residential investment good market)

3. Calibration

Based on Kiyotaki and Moore (1997) discount factor for unconstrained households $\beta^n$ is set equal to conventional 0.99 while the discount factor for credit constrained households $\beta^c$ to 0.96. Following Campbell and Cocco (2005) we set $\theta = 1.2$ and $\gamma = 2$. The value of $m$ in the baseline case is set equal to 0.8 which is the average loan-to-value ratio for conventional mortgages in US for years 1990-2000 according to Monthly Interest Rate Survey of Federal Housing Finance agency. The second considered value of $m$ is set to 0.93 reflecting the rapid liberalization of mortgage conditions which happened from 2000 to 2006. The third value is set to 0.85, reflecting the post crisis tightening of the mortgage conditions (value again obtained from Monthly Interest Rate Survey as average for years 2007-2010). Depreciation rate for physical capital and housing, as well as capital and labor shares in the production of intermediate goods, shares of each of the intermediate goods in the production of residential investment good and composite consumption goods and shares of residential investment good and land in the production of housing are set equal to the corresponding values from Davis and Heatcote (2005). In the benchmark case relative size of unconstrained households $g$ is set equal to 2, implying that there twice as many unconstrained households in the economy as constrained ones. Later on sensitivity analysis is performed with respect to this parameter. The remaining parameters, that is $a, \bar{z}, d, \rho, \sigma$ and $\chi$ were estimated by means of GMM estimation of equation (3), equation(18) and equation (19) and the following data for period 1987-2009: a) Freddie Mac Conventional Mortgage Home Price Index divided by NIPA price index for Personal Consumption Expenditure; b) Wharton Regulation Index from Saiz(2010); c) multi-factor productivity in construction sector from EU-KLEMS database d) multi-factor productivity in manufacturing sector from EU-KLEMS database e) new housing units completed from US Census Bureau. All the values of parameters are displayed in Table 1


After deriving steady state and log-linearizing the model the model is used to investigate how shocks affecting housing price which were present in the US economy over the period 1995-2010 affected consumption and housing allocations, prices and welfare of separate
groups as well as aggregate welfare.

First the variance of residuals of stochastic shocks represented in this model by productivity shocks in manufacturing good production sector and housing production sector (variables $z_t, \eta_t$) is calculated using the corresponding multi-factor productivity time series for 1995-2010 from EU-KLEMS database. Next, to generate positive income shock, which was characteristic for the US over the first 11 years (that is 1995-2006) we shift the intercept of the process for productivity shock in manufacturing good production that is parameter $z_t$. We shift it by the magnitude which is enough to generate 10% higher wage in the new steady state which is consistent with evolution of wages in the US over the considered period. Also for the first 11 periods of the simulation the loan-to-value ratio $m$ shifts from 0.8 to 0.93 reflecting the rapid liberalization in the credit market. The last 4 periods of the simulation are reflecting the period of negative shocks in the actual US economy which happened from 2007. For those periods we use lower value of $z_t$ enough to generate decrease in wage by 7% and we also shift down the loan-to-value ratio to 0.85.

The transition paths of the main endogenous variables are represented in Figure 1. We can see that housing price as well as wage and capital accumulation increases until period 6 and starts to decline as the effect of positive shocks dies out and the expectations of negative shocks starts to play a role. While the composite good consumption as well as housing consumption of unconstrained households first increases and then adjusts down in expectation of negative shocks, the composite good consumption and housing consumption of constrained households decreases during first period and only then increases reacting to positive shocks.

If we take the entire period it is visible that aggregate consumption of composite good aggregate consumption of housing and aggregate lifetime utility increase, which implies that the effect of observed positive shocks in total dominates the effect of negative ones. In particular, the aggregate consumption increases by 5.2%, the aggregate housing consumption increases by around 0.5% and aggregate lifetime utility increases by 1.42%.

**References**


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**APPENDIX**

**Figure 1:** Transitional dynamics of main endogenous variables (increase in productivity in the intermediate good production sector)
### Table 1: Parameter values

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^c$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\beta^{uc}$</td>
<td>0.99</td>
</tr>
<tr>
<td>$m$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.132</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.309</td>
</tr>
</tbody>
</table>
$\alpha_s \quad 0.237$
$\alpha_{co} \quad 0.0307$
$\beta_{co} \quad 0.2696$
$\sigma_{res} \quad 0.4697$
$\beta_{res} \quad 0.2382$
$\varepsilon \quad 0.894$
$a \quad 0.3$
$d \quad -0.4$
$\rho \quad 0.3$
$g \quad 2$
$\theta \quad 1.2$
$\gamma \quad 2$
$\delta_h \quad 0.014$
$\delta_k \quad 0.057$
$\chi \quad 0.450$
$\varphi \quad 0.322$
$\sigma \quad 0.3$
$\tau \quad 0.7$
Table 2: Steady state values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
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<td>$c_u$</td>
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