SURVIVAL DATA ANALYSIS BY GENERALIZED ENTROPY OPTIMIZATION METHODS

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Abstract

Entropy Optimization Methods (EOM) have important applications, especially in statistics, economy, engineering and etc. There are several examples in the literature that known statistical distributions do not conform to statistical data, however the entropy optimization distributions do conform well. It is known that all statistical distributions can be obtained as the MaxEnt distribution and Entropy Optimization Distribution (EOD) especially as Generalized Entropy Optimization Distribution (GEOD) more exactly represents the given statistical data.

In this paper, survival data analysis is fulfilled by applying Generalized Entropy Optimization Methods (GEOM). GEOM have suggested distributions in the form of the MinMaxEnt, the MaxMaxEnt which are closest and furthest to statistical data in the sense of information theory respectively.

In this research, the data of male patients with localized cancer of a rectum diagnosed in Connecticut from 1935 to 1944 is considered and the results are acquired by using statistical software R and MATLAB. The performances of GEOD are established by Chi – Square, Root Mean Square Error (RMSE) and Information criteria.

Key words: Censored Observation, Generalized Entropy Optimization Methods, MaxEnt, MinMaxEnt, MaxMaxEnt distributions.

JEL Code: C16, C19, D89

Introduction

Entropy Optimization Methods (EOM) have important applications, especially in statistics, economy, engineering and so on. There are several examples in the literature that known statistical distributions do not conform to statistical data; however, the entropy optimization distributions conform well. Generalized Entropy Optimization Methods (GEOM) have suggested distributions in the form of the MinMaxEnt, the MaxMaxEnt which are closest or furthest to statistical data in the sense of information theory (Shamilov, 2006), (Shamilov, 2007). For this reason, GEOM can be more successfully applied in Survival Data Analysis.

Different aspects and methods of investigations of survival data analysis are considered in (Kaminski & Geisler, 2012), Reingold and et. al.(2012), Wang and et. al. (2012), (Ebrahimi, 2000), Guyot and et. al. (2012) and Joly and et. al. (2012).

In particular in the paper (Ebrahimi, 2000) it is investigated several problems of hazard rate function estimation based on the maximum entropy principle. The potential applications include developing several classes of the maximum entropy distributions which can be used to model different data-generating distributions that satisfy certain information constraints on the hazard rate function.

In order to represent the results of our investigations, we give some auxiliary concepts and facts first.

1. Survival Analysis

Survival Time: can be defined broadly as the time to the occurrence of a given event. This event can be the development of a disease, response to a treatment, relapse or death (Lee & Wang, 2003).

Censoring: The techniques for reducing experimental time are known as censoring. In survival analysis, the observations are lifetimes which can be indefinitely long. So quite often the experiment is so designed that the time required for collecting the data is reduced to manageable levels.

Let T be a continuous, non-negative valued random variable representing the lifetime of a unit. This is the time for which an individual (or unit) carries out its appointed task satisfactorily and then passes into ‘‘ failed’’ or ‘‘dead’’ state thereafter (Deshpande & Purohit, 2005).

2. Generalized Entropy Optimization Methods (GEOM)

Entropy Optimization Problem (EOP) (Kapur & Kesavan, 1992) and Generalized Entropy Optimization problem (GEOP) (Shamilov, 2007) can be formulated in the following form.

EOP: Let $f^{(0)}(x)$ be given probability density function (p.d.f.) of random variable X, L be an entropy optimization measure and g be a given moment vector function generating m moment constraints. It is required to obtain the distribution corresponding to g, which gives extreme value to L.

GEOP: Let $f^{(0)}(x)$ be given probability density function of random variable X, L be an entropy optimization measure and K be a set of given moment vector functions. It is required to choose moment vector functions $g^{(1)}, g^{(2)} \in K$ such that $g^{(1)}$ defines entropy optimization distribution $f^{(1)}(x)$ closest to $f^{(0)}(x)$, $g^{(2)}$ defines entropy optimization distribution $f^{(2)}(x)$ furthest from $f^{(0)}(x)$ with respect to entropy optimization measure L. If L is taken as Shannon entropy measure, then $f^{(1)}(x)$ is called the MinMaxEnt distribution, and $f^{(2)}(x)$ is called the MaxMaxEnt distribution (Shamilov, 2007).

The method of solving GEOP is called as GEOM.

2.1 MaxEnt Functional

The problem of maximizing entropy function

$$H(p) = -\sum_{i=1}^{n} p_i \ln p_i \ , \qquad p = (p_1, \cdots, p_n) \qquad (1)$$

subject to constraints

$$\sum_{i=1}^{n} p_i g_j(x_i) = \mu_j \ , \qquad j = 0,1,2,\cdots,m \qquad (2)$$

where $\mu_0 = 1$, $g_0(x) = 1$, $p_i \geq 0$, $i = 1,2,\cdots,n$; $m+1 < n$,

has solution

$$p_i = e^{-\sum_{j=0}^{m} \lambda_j g_j(x_i)}, \quad i = 1,2,\cdots,n \qquad (3)$$

where λ_j $(j = 0,1,2,\cdots,m)$ are Lagrange multipliers. In the literature, there have been numerous studies that have calculated these multipliers (Shamilov, 2006). In this study, we use the MATLAB program to calculate Lagrange multipliers.

If Eq. (3) is substituted in Eq.(1), the maximum entropy value is obtained:

$$H_{max}(p) = -\sum_{i=1}^{n} e^{-\sum_{j=0}^{m} \lambda_j g_j(x_i)} \left(-\sum_{j=0}^{m} \lambda_j g_j(x_i)\right) = \sum_{j=0}^{m} \lambda_j \mu_j \ . \qquad (4)$$

If distribution $p^{(0)} = \left(p_1^{(0)}, \cdots, p_n^{(0)}\right)$ is calculated from the data, the moment vector value $\mu = (1, \mu_1, \cdots \mu_m)$ can be obtained for each moment vector function $g(x) = \left(1,\ g_1(x), \cdots, g_m(x)\right)$. Thus, H_{max} is considered as a functional of $g(x)$ and called the MaxEnt functional. Therefore, we use the notation $U(g)$ to denote the maximum value of H corresponding to $g(x) = \left(1,\ g_1(x), \cdots, g_m(x)\right)$.

2.2. MinMaxEnt and MaxMaxEnt Distributions

Let K be the compact set of moment vector functions $g(x)$. $U(g)$ reaches its least and greatest values in this compact set, because of its continuity property. For this reason,

$$\min_{g \in K} U(g) = U\left(g^{(1)}\right); \qquad \max_{g \in K} U(g) = U\left(g^{(2)}\right).$$

Consequently,

$$U\left(g^{(1)}\right) \leq U\left(g^{(2)}\right).$$

Distributions $p^{(1)} = \left(p_1^{(1)}, \cdots, p_n^{(1)}\right)$ and $p^{(2)} = \left(p_1^{(2)}, \cdots, p_n^{(2)}\right)$ corresponding to the $g^{(1)}(x)$ and $g^{(2)}(x)$, respectively, are called MinMaxEnt and MaxMaxEnt distributions (Shamilov, 2006).

3. Application of MinMaxEnt and MaxMaxEnt Methods to Survival Data

In the present research, the data of male patients with localized cancer of a rectum diagnosed in Connecticut from 1935 to 1944 given in Tab. 1 is considered (Lee & Wang, 2003).

Tab. 1: The data of male patients with localized cancer of a rectum diagnosed in Connecticut from 1935 to 1944 and observation probabilities, corrected probabilites

Survival Time (year) t	Number of patients surviving at beginning of interval n_i	n_i'	Number of patients dying in interval d_i	Number of patients censoring in interval c_i	Observation probabilities p_i	Corrected probabilities p_i^*
0 – 1	388	336	167	2	0.4970	0.5030
1 – 2	219	167	45	1	0.1339	0.1369
2 – 3	173	121	45	1	0.1339	0.1369
3 – 4	127	75	19	0	0.0565	0.0565
4 – 5	108	56	17	0	0.0506	0.0506
5 – 6	91	39	11	1	0.0327	0.0357
6 – 7	79	27	8	0	0.0238	0.0238
7 – 8	71	19	5	0	0.0149	0.0149
8 – 9	66	14	6	1	0.0179	0.0208
9 – 10	59	7	7	0	0.0208	0.0208

Source: In here, n_i' denotes that number of patients surviving except for the presence of censoring from the planning patients 52 individuals at beginning of interval

In this investigation, the experiment is planned for 388 numbers of patients surviving at beginning of interval but the presence of censoring from the planning patients 52 individuals stay out the experiment. This situation is taken into account in Tab. 1.

It should be noted that, the presence of censoring in the survival times leads to a situation where the sum of observation probabilities stands less than 1 for the survival data. For this reason, in solving many problems, it is required to supplement the sum of observation probabilities up to 1. Since the sum of observation probabilities p_i in Tab. 1 is 0.9821, according to the number of censoring, supplementary probability

$$1 - 0.9821 = 0.0179$$

is uniformly distributed to each censoring data and corrected probabilities p_i^* are obtained.

Let $K_0 = \{g_1, \cdots, g_r\}$ be the set of characterizing moment vector functions and all combinations of r elements of K_0 taken m elements at a time be $K_{0,m}$. We note that, each element of $K_{0,m}$ is vector g with mcomponents.

Solving the MinMaxEnt and the MaxMaxEnt problems require to find vector functions $\left(g_0, g^{(1)}(x)\right)$, $\left(g_0, g^{(2)}(x)\right)$, where $g_0(x) = 1$, $g^{(1)} \in K_{0,m}$, $g^{(2)} \in K_{0,m}$ which give minimum and maximum values to $U(g)$ accordingly with respect to Shannon entropy measure. It should be noted that $U(g)$ reaches its minimum (maximum) value subject to constraints generated by vector functions $g_0(x)$ and all $m-$dimensional vector functions $g(x)$, $g \in K_{0,m}$. In other words, minimum (maximum) value of $U(g)$ is least (greatest) value of values H_{max} corresponding to g(x), $g \in K_{0,m}$. If $\left(g_0, g^{(1)}\right)\left(\left(g_0, g^{(2)}(x)\right)\right)$ gives the minimum (maximum) value to $U(g)$ then distribution $p^{(1)} = \left(p_1^{(1)}, \cdots, p_n^{(1)}\right)$ $\left(p^{(2)} = \left(p_1^{(2)}, \cdots, p_n^{(2)}\right)\right)$ corresponding to $\left(g_0, g^{(1)}\right)$ $\left(\left(g_0, g^{(2)}(x)\right)\right)$ is called the MinMaxEnt (the MaxMaxEnt) distribution.

It is easy to prove the following result.

Theorem: If by $(MaxMaxEnt)_m$ $((MinMaxEnt)_m)$ denote the MaxMaxEnt (the MinMaxEnt) distribution corresponding to m moment conditions, then inequality

$$H\left((MaxMaxEnt)_{m_1}\right) > H\left((MaxMaxEnt)_{m_2}\right)$$

$$\left(H\left((MinMaxEnt)_{m_1}\right) > H\left((MinMaxEnt)_{m_2}\right)\right)$$

is fulfilled, when $m_1 > m_2$. In other words, entropy value of the MaxMaxEnt (the MinMaxEnt) distribution depending on the number m of moment conditions decreases.

Of course, the inequality $H((MaxMaxEnt)_m) > H((MinMaxEnt)_m)$ takes place naturally. This result shows that both distributions can be applied in solving proper problems in survival data analysis.

In our investigation as characterizing moment vector functions $g_1(x) = x$, $g_2(x) = x^2$, $g_3(x) = ln\, x$, $g_4(x) = (ln\, x)^2$, $g_5(x) = ln(1 + x^2)$ are chosen. Consequently, $K_0 = \{g_1, \cdots, g_5\}$. For example, if $m = 3$ then

$$(g_0,\ g^{(1)}) = (1, x, x^2, (\ln x)^2)\ , \qquad g^{(1)} \in K_{0,3}$$

gives the least value to U(g) and

$$(g_0, g^{(2)}) = (1, x, x^2, ln(1 + x^2))\ , \qquad g^{(2)} \in K_{0,3}$$

gives the greatest value to U(g).

The MaxEnt distributions corresponding to $(g_0,\ g)$, $g_0(x) = 1$, $g \in K_{0,m}$ $m = 3{,}4$ and H_{max} values are shown in Tab. 2 - 3. By virtue of these tables also are obtained $(MinMaxEnt)_m$, $(MaxMaxEnt)_m$, $m = 1{,}2\ , \cdots ,4$ distributions which are shown in Tab. 4, 5. It should be noted that the MaxEnt, $(MinMaxEnt)_m$, $(MaxMaxEnt)_m$, $m = 1{,}2, \cdots ,4$ distributions for the investigated data are determined by using MATLAB.

In order to obtain the performance of the mentioned distributions, we use various criteria as Root Mean Square Error (RMSE), Chi-Square and entropy values of distributions. The acquired results are demonstrated in Tab. 6, 7.

In the sense of RMSE criteria each $(MinMaxEnt)_m$ ($m = 1{,}2{,}3$) distribution is better than corresponding $(MaxMaxEnt)_m$ ($m = 1{,}2{,}3$) distribution but, $(MaxMaxEnt)_4$ distribution is more suitable for statistical data than $(MinMaxEnt)_4$ distribution.

When the MaxEnt distribution corresponding to $(g_0,\ g)$, $g_0(x) = 1$, $g \in K_{0,1}$ and H_{max} values are calculated it is seen that the MinMaxEnt (the MaxMaxEnt) distribution is realized by vector function $(g_0, g_3) = (1, lnx)$ $((g_0, g_2) = (1, x^2))$ and $H((MinMaxEnt)_1) = 2.3840$ $(H((MaxMaxEnt)_1) = 2.6554)$.

If the MaxEnt distribution corresponding to $(g_0,\ g)$, $g_0(x) = 1$, $g \in K_{0,2}$ and H_{max} values are calculated then the MinMaxEnt (the MaxMaxEnt) distribution is realized by vector function $(g_0, g_1, g_3) = (1, x, lnx)$ $((g_0, g_2, g_4) = (1, x^2, (ln\, x)^2))$ and $H((MinMaxEnt)_1) = 2.3799$ $(H((MaxMaxEnt)_1) = 2.6416)$.

Tab. 2: The MaxEnt distribution corresponding to (g_0, g), $g_0(x) = 1$, $g \in K_{0,3}$ and H_{max} values

(g_0, g)	(g_0, g_1, g_2, g_3)	(g_0, g_1, g_2, g_4)	(g_0, g_1, g_2, g_5)	(g_0, g_1, g_3, g_4)	(g_0, g_1, g_3, g_5)
MaxEnt Dist.	0.4963	**0.5011**	**0.4760**	0.4997	0.5007
	0.1707	**0.1612**	**0.2004**	0.1620	0.1592
	0.0976	**0.0996**	**0.0992**	0.0976	0.0980
	0.0652	**0.0679**	**0.0607**	0.0678	0.0687
	0.0472	**0.0485**	**0.0427**	0.0500	0.0507
	0.0360	**0.0360**	**0.0328**	0.0380	0.0383
	0.0284	**0.0278**	**0.0268**	0.0294	0.0294
	0.0231	**0.0224**	**0.0229**	0.0230	0.0229
	0.0192	**0.0189**	**0.0202**	0.0182	0.0179
	0.0163	**0.0167**	**0.0182**	0.0144	0.0141
H_{max}	2.3798	**2.3761**	**2.3903**	2.3794	2.3790
(g_0, g)	(g_0, g_2, g_3, g_5)	(g_0, g_2, g_3, g_5)	(g_0, g_2, g_3, g_5)	(g_0, g_2, g_3, g_5)	(g_0, g_2, g_3, g_5)
MaxEnt Dist.	0.4994	0.4986	0.5001	0.4972	0.5012
	0.1620	0.1664	0.1631	0.1687	0.1562
	0.0973	0.0965	0.0962	0.0952	0.0998
	0.0679	0.0659	0.0666	0.0651	0.0705
	0.0504	0.0485	0.0494	0.0486	0.0514
	0.0384	0.0373	0.0380	0.0379	0.0383
	0.0297	0.0293	0.0297	0.0300	0.0291
	0.0231	0.0234	0.0235	0.0238	0.0224
	0.0179	0.0189	0.0186	0.0188	0.0174
	0.0140	0.0153	0.0148	0.0147	0.0138
H_{max}	2.3801	2.3802	2.3802	2.3822	2.3779

From Tab. 2 it is seen that the MinMaxEnt (the MaxMaxEnt) distribution is realized by vector function $(g_0, g_1, g_2, g_4) = (1, x, x^2, (\ln x)^2)$ $\left((g_0, g_1, g_2, g_5) = (1, x, x^2, \ln(1 + x^2))\right)$ and $H((MinMaxEnt)_1) = 2.3761$ $(H((MaxMaxEnt)_1) = 2.3903)$

Tab. 3: The MaxEnt distribution corresponding to (g_0, g), $g_0(x) = 1$, $g \in K_{0,4}$ and H_{max} values

(g_0, g)	$(g_0, g_1, g_2, g_3, g_4)$	$(g_0, g_1, g_2, g_3, g_5)$	$(g_0, g_1, g_2, g_4, g_5)$	$(g_0, g_1, g_3, g_4, g_5)$	$(g_0, g_2, g_3, g_4, g_5)$
MaxEnt Dist.	**0.5024**	0.5025	0.5024	**0.5026**	0.5025
	0.1442	0.1438	0.1439	**0.1428**	0.1433
	0.1151	0.1157	0.1155	**0.1176**	0.1166
	0.0780	0.0781	0.0781	**0.0780**	0.0781
	0.0488	0.0485	0.0486	**0.0473**	0.0480
	0.0312	0.0310	0.0310	**0.0304**	0.0307
	0.0220	0.0220	0.0220	**0.0221**	0.0220
	0.0180	0.0182	0.0181	**0.0188**	0.0184
	0.0180	0.0181	0.0181	**0.0187**	0.0184
	0.0224	0.0222	0.0223	**0.0217**	0.0220
H_{max}	**2.3687**	2.3686	2.3686	**2.3682**	2.3684

From Tab. 3 it is seen that the MinMaxEnt (the MaxMaxEnt) distribution is realized by vector function $(g_0, g_1, g_3, g_4, g_5) = (1, x, \ln x, (\ln x)^2, \ln(1 + x^2))$ $\left((g_0, g_1, g_2, g_3, g_4) = (1, x, x^2, \ln x, (\ln x)^2)\right)$ and $H((MinMaxEnt)_1) = 2.3682$ $(H((MaxMaxEnt)_1) = 2.3687)$.

Tab. 4: Distributions of $(MinMaxEnt)_m$, $m = 1, \cdots 4$

T	d_i	c_i	p_i^*	$(MinMaxEnt)_1$	$(MinMaxEnt)_2$	$(MinMaxEnt)_3$	$(MinMaxEnt)_4$
1	167	2	0.5030	0.5115	0.4948	0.5011	0.5026
2	45	1	0.1369	0.1541	0.1762	0.1612	0.1428
3	45	1	0.1369	0.0882	0.1013	0.0996	0.1176
4	19	0	0.0565	0.0611	0.0672	0.0679	0.0780
5	17	0	0.0506	0.0464	0.0478	0.0485	0.0473
6	11	1	0.0357	0.0373	0.0354	0.0360	0.0304
7	8	0	0.0238	0.0311	0.0269	0.0278	0.0221
8	5	0	0.0149	0.0266	0.0209	0.0224	0.0188
9	6	1	0.0208	0.0232	0.0165	0.0189	0.0187
10	7	0	0.0208	0.0205	0.0131	0.0167	0.0217

Tab. 5: Distributions of $(MaxMaxEnt)_m$, $m = 1, \cdots 4$

T	d_i	c_i	p_i^*	$(MaxMaxEnt)_1$	$(MaxMaxEnt)_2$	$(MaxMaxEnt)_3$	$(MaxMaxEnt)_4$
1	167	2	0.5030	0.2566	0.2490	0.4760	0.5024
2	45	1	0.1369	0.2308	0.2811	0.2004	0.1442
3	45	1	0.1369	0.1867	0.1864	0.0992	0.1151
4	19	0	0.0565	0.1359	0.1160	0.0607	0.0780
5	17	0	0.0506	0.0889	0.0709	0.0427	0.0488
6	11	1	0.0357	0.0523	0.0428	0.0328	0.0312
7	8	0	0.0238	0.0277	0.0255	0.0268	0.0220
8	5	0	0.0149	0.0132	0.0149	0.0229	0.0180
9	6	1	0.0208	0.0056	0.0086	0.0202	0.0180
10	7	0	0.0208	0.0022	0.0048	0.0182	0.0224

Tab. 6: The obtained results for $(MinMaxEnt)_m$, $m = 1, 2, \cdots, 4$

Distribution of MinMaxEnt	H	Calculated value of Chi – Square	Table value of Chi – Square	RMSE
$(MinMaxEnt)_1$	2.3840	0.7740	$\chi^2_{8,\alpha} = 15.51$	0.1680
$(MinMaxEnt)_2$	2.3799	0.6512	$\chi^2_{7,\alpha} = 14.07$	0.1841
$(MinMaxEnt)_3$	2.3761	0.5025	$\chi^2_{6,\alpha} = 12.59$	0.1374
$(MinMaxEnt)_4$	2.3682	0.2458	$\chi^2_{5,\alpha} = 11.07$	0.0984

Tab. 7: The obtained results for $(MaxMaxEnt)_m$, $m = 1, 2, \cdots, 4$

Distribution of MaxMaxEnt	H	Calculated value of Chi – Square	Table value of Chi – Square	RMSE
$(MaxMaxEnt)_1$	2.6554	11.6661	$\chi^2_{8,\alpha} = 15.51$	1.9600
$(MaxMaxEnt)_2$	2.6416	9.5441	$\chi^2_{7,\alpha} = 14.07$	0.8416
$(MaxMaxEnt)_3$	2.3903	0.8717	$\chi^2_{6,\alpha} = 12.59$	0.1465
$(MaxMaxEnt)_4$	2.3687	0.2598	$\chi^2_{5,\alpha} = 11.07$	0.0981

Conclusion

In this study, it is shown that $(MinMaxEnt)_4$ and $(MaxMaxEnt)_4$ distributions successfully represent Survival Data. $(MaxMaxEnt)_4$ distribution is more suitable for statistical data than $(MinMaxEnt)_4$ distribution in the sense of RMSE criteria while $(MinMaxEnt)_4$ is more suitable than $(MaxMaxEnt)_4$ distribution in the sense of information measure. Furthermore, our investigation indicates that GEOM in survival data analysis yields reasonable results.

References

[1] Deshpande & Purohit. (2005), *Life Time Data: Statistical Models and Methods.* India: Series on Quality, Reliability and Engineering Statistics.

[2] Ebrahimi, (2000). The Maximum Entropy Method for Lifetime Distributions. *Sankhyā:* (Vol. 62, No. 2) 236-243.

[3] Guyot, Ades, Ouwens & Welton, (2012). Enhanced Secondary Analysis of Survival Data: Reconstructing The Data From Published Kaplan-Meier Survival Curves. *BMC Medical Research Methodology* (Vol. 12, Isssue.9).

[4] Joly, Gerds, Qvist, Commenges & Keiding (2012). Estimating Survival of Dental Fillings on The Basis of Interval-Censored Data and Multi-State Models. *Statistics in Medicine* (Vol. 31 Issue 11-12).

[5] Kaminski & Geisler (2012). Survival Analysis of Faculty Retention in Science and Engineering by Gender. *Science* (Vol. 335 no. 6070) 864-866.

[6] Kapur & Kesavan. (1992), *Entropy Optimization Principles with Applications.*

[7] Lee & Wang. (2003), *Statistical Methods for Survival Data Analysis*. Oklahoma: Wiley-Interscience.

[8] Papoulis & Pillai. (2002), *Probablitiy, Random Variables and Stochastic Processes.*

[9] Reingold, Reichle, Glaholt & Sheridan (2012). Direct Lexical Control of Eye Movements in Reading: Evidence From A Survival Analysis of Fixation Durations. *Cognitive Psychology* (Vol. 65, Issue 2) 177–206.

[10] Shamilov (2006). A Development of Entropy Optimization Methods. *WSEAS Transactions on Mathematics* (Vol. 5, Issue 5) 568-575.

[11] Shamilov (2007). Generalized Entropy Optimization Problems And The Existence of Their Solutions. *Physica A : Statistical Mechanics and its Applications* (382(2)) 465-472.

[12] Shamilov, Giriftinoglu, Usta & Mert Kantar (2008). A New Concept of Relative Suitability of Moment Function Sets. *Applied Mathematics and Computation* (206) 521-529.

[13] Shamilov. (2009), Entropy*, Information and Entropy Optimization*, Eskisehir: T.C. Anadolu University Publication.

[14] Shamilov (2010). Generalized entropy optimization problems with finite moment functions sets. *Journal of Statistics and Management Systems* (Vol. 13, Issue 3) 595-603.

[15] Wang Dai & Fu (2012). Accelerated Failure Time Models For Censored Survival Data Under Referral Bias. *Biostatistics* (Vol. 14, Issue 2) 313-326.

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