# MEASUREMENT OF INEQUALITY IN SLOVAKIA

**Ľubica Sipková – Juraj Sipko**

**Abstract**

The main goal of the paper is to analyse income inequality in the regions of the Slovak Republic by the methodology by Tarsitano using Kappa generalized quantile function.

Various inequality dimensions are characterized and geographically compared by relative inequality measures. In order to measure income inequality by specific measures such as the Gini coefficient, DeVergottini rate, Bonferroni and Tarsitano measures of inequality, have been used the parametric approach based on quantile models. The four-parameter Kappa generalized quantile distribution is flexible enough for regional - comparative population modeling, which is functional for inequality measures.

Based on quantitative analysis by using the quantile methods the paper concluded that the Gini, DeVergottini, Bonferroni and Tarsitano measures of inequality, which could be defined by one generalized formula proposed by Tarsitano, are analogous in regional studies. To condense information on inequality of wages of employees of the Slovak Republic and its regions, official individual observations were retained in calculations of Tarsitano's generalized inequality measure based on rankits and displayed in Q-Q plots.

Paper came to the conclusion that the same order of geographical regions by the level of income inequality could be reached with using the inequality indexes defined by different type of quantile models.

**Key words:** income inequality, Gini coefficient, quantile probability function, Tarsitano's inequality generalized formula, gross wages

**JEL Code:** J31, C14, C46

## Introduction

The global financial crisis and recession intensified the discussion on growing income inequality around the globe. Uneven, fragile and uncertain economic growth in the world economy, but mainly in some EU member countries pays attention of academia researchers and policy-makers how to ensure the social stability.

A large number of scientific studies, with the support of detailed quantitative analysis are contradictory in its conclusions on economic and social processes taking place in the background of growing inequality and poverty in the current world. According to Ravallion (2003) the sources of different conclusions are unclear, ambiguous or inconsistent specification of the analysis concept, various definitions of analyzed data and use of different methodologies. Loungani (2003) defines three basic concepts of measuring inequality: cross-country inequality, within-country inequality and global inequality concepts. Which concept of inequality measurement is most appropriate depends on the intended use of the analysis results. Comparison of income inequality based on different concepts is problematic.

Income is not the only widely used measure of evaluation standard of living. Household consumption, life expectancy and educational level are subjects to different concepts of measuring and comparing living conditions.

In the paper, measuring of a within-country inequality in quality of life is reduced to measuring of income inequality. The spatial analysis of income inequality in the paper is based on the so-called primary income of individuals. Gross wages of an employee is the individual's income before the application of taxes, levies and without the addition of any benefits.

Analysis in the paper follows the results of income inequality analysis in eight NUTS III regions of the Slovak Republic, which were disclosed in the article by Sipko and Sipková (2013). Utilization of the methods in the article is based on Galton’s quantiles and Order statistics theory (see e.g. David & Nagaraja, 2003). The mentioned article (Sipko & Sipková, 2013) and the paper provide results of application of the quantile approach to analyze relative income inequality and income inequality quasi-orderings presented by Amartya Sen (1973, 1997). The standard statistical approach has been used, that considers income distribution as the outcome of a stochastic process. The results of modelling by generalized Kappa distribution defined by Hosking (1994) and the least squares estimate of the slope in Q-Q graph as a measure of the inequality in the observed income distribution are presented in the third chapter of the mentioned article.

The Gini coefficient is a measure of inequality in comparison to the uniform distribution, De Vergottini rate to the exponential shape and Bonferroni to the basic quantile form of reflected-exponential distribution. The *V(inequality)* was calculated in accordance with the Tarsitano approach in comparison with the Kappa probabilistic model. A greater Tarsitano value of inequality *V(inequality)* means a greater slope of the line of the Q-Q graphical representations and a greater inequality of income distribution. In the applied parametric approach, each calculated inequality measure of income distribution is based on a comparison to a different theoretical probability distribution model.

On the basis of the Q-Q graphical representation comparison ranking of Kappa models against the empirical shape adapted according to Tarsitano, among other statements we concluded in the mentioned article (Sipko and Sipkova, 2013), that “the three reported measures of income inequality do not mutually correspond” in regional studies; and “each compares to a different model shape and reflects a different aspect of inequality”.

Because of the Tarsitano summarization of results of probabilistic modeling of income inequality probability distribution by means of Q-Q graphs is very synoptic and it is an interesting solution and marks progress in studies of income inequality, we father elaborated his methodological approach and finally found the method, that all the corresponding inequality measures could agree in regional comparison by using Tarsitano theory.

The applications in the paper were accomplished with technical support of computer programs in STATA, DAD, STATGRAPHICS and EXCEL.

# 1 Data source and methodology

The data source for all analysis in the paper is a Information System of Labour Costs (ISCP), survey included in the Programme of State Statistical Surveys, which is provided by TREXIMA, Ltd. for the purposes of the Ministry of Labour, Social Affairs and Family of the Slovak Republic. The databases for analyses were created by simple random sampling in the range of one percent of all employees of the Slovak Republic from the survey ISCP (in the range of 9,900 employees in 2010).

Method of calculation and methodological notes on the content of surveyed indicators with guidance notes are available at the official website of TREXIMA, Ltd. together with the necessary details, classifications and methodological aids for the organizations providing information about employees. Description of the background empirical income distribution could be found in the article by Sipko and Sipková (2013).

Eight regions of the Slovak Republic are eight categories of variable *KRAJ* in accordance with the NUTS III classification in the following designation: 1 – Bratislavský (BA), 2 – Trnavský (TT), 3 – Trenčiansky (TN), 4 – Nitriansky (NR), 5 – Žilinský (ZA), 6 – Banskobystrický (BB), 7 – Prešovský (PO), 8 – Košický (KE).

Applied quantile modeling methods are broadly described in Warren Gilchrist's: Modelling with quantile distribution functions (Gilchrist, 2000), and methods of relative inequality evaluation in Agostiano Tarsitano's: A new Q-Q plot and its application to income data (Tarsitano, 2006).

The four-parameter Kappa distribution by Hosking (1994), collaborated as a linear-exponential transformation of Gumbel-Exponential shape is defined by its quantile function:

$$Q(p,\lambda) = \lambda_1 + \frac{\lambda_2}{\lambda_3}\left[1-\left(\frac{1-p^{\lambda_4}}{\lambda_4}\right)^{\lambda_3}\right]; \quad 0 < p \le 1, \lambda_2 > 0 \qquad (1)$$

We applied its reduced version in standardized form without a location parameter $\lambda_1$ and a scale parameter $\lambda_2$, with only two exponential parameters $\lambda_3$, $\lambda_4$, that determine the shape of the standardized form of the Kappa quantile function (Tarsitano, 2006, p. 12):

$$est\ E(X_{i:n}) = est\ W_{i:n} = w_{i:n}\left[p_i^{-\lambda_4} - 1\right]^{-\lambda_3},\ i = 1,\ 2 \ldots n \qquad (2)$$

where $0 \le p_i \le 1$ is point estimate of probability by the plotting position suggested by Landwehr (with 0.35 and 0.65 proportions).

The expected value of *i*-th recipient income in "fair equal" income distribution, considered by the quantile income model $Q(p)$, is point estimate of the expected value of the distribution of the *i*-th order statistic $X_{i:n}$, therefore *i*-th value of the rankit of the quantile income model $w_{i:n}$. The point estimate of the expected value of the rankit $W_n$ of the income distribution $Q(p)$ could be defined as:

$$est\ W_n = w_n = n^{-1}\sum_{i=1}^{n} w_{i:n},\ i = 1,\ 2 \ldots n \qquad (3)$$

The expected value of the rankit of the quantile income distribution $\mu_n$ could be estimated by *n*-ordered income values:

$$est\ \mu_n = n^{-1}\sum_{i=1}^{n} x_{i:n},\ i = 1,\ 2 \ldots n \qquad (4)$$

Degree of dissatisfaction of *i*-th income recipient by degree of income inequality, respectively contribution of an individual relative inequality to the total relative income inequality could be measured by the product of two factors in the formula (5).

$$V_{i:n} = R_{i:n} \cdot \left( \frac{X_{i:n}}{\mu_n} \right), \ i = 1, 2 \ldots n, \tag{5}$$

The point estimate of the first factor $R_{i:n}$ is normalized expected income of *i*-th order statistic $X_{i:n}$, computed according income distribution model $Q(p)$:

$$est\, R_{i:n} = r_{i:n} = \frac{w_{i:n} - w_n}{w_{n:n} - w_n}, \ i = 1, 2 \ldots n \tag{6}$$

where $w_{i:n}$ are point estimates of rankit of the chosen form of the quantile income model $Q(p_i)$, $i = 1, 2 \ldots n$ and $w_n$ is the point estimate of the expected value of the rankit.

The relative measure of inequality, based in expected value of order statistics of "fair equal" income distribution defined by the quantile income model $Q(p)$, could be obtained as the arithmetic average of all „dissatisfaction" of each individual income distribution expectations.

According to the chosen type of income distribution $Q(p)$ is the value of the relative inequality measure:

$$est\, V_n = n^{-1} \sum_{i=1}^{n} v_{i:n} = \sum_{i=1}^{n} r_{i:n} \left( \frac{x_{i:n}}{t} \right); \quad t = n \cdot est\, \mu_n = \sum_{i=1}^{n} x_{i:n}; \ \sum_{i=1}^{n} r_{i:n} = 0 \tag{7}$$

Tarsitano's inequality measures based on rankit of Kappa normalized shape actualized in consequent years had been proposed for yearly income inequality comparison in Italia in Q-Q plots using the equation (Tarsitano, 2006, p. 14):

$$V_n(\lambda_3, \lambda_4) = \sum_{i=1}^{n} \left[ \frac{\left[p_i^{-\lambda_4} - 1\right]^{-\lambda_3} - w_n(\lambda_3, \lambda_4)}{\left[p_n^{-\lambda_4} - 1\right]^{-\lambda_3} - w_n(\lambda_3, \lambda_4)} \right] \frac{X_{i:n}}{T}, \ i = 1, 2 \ldots n, \ T = n\mu_n; \quad w_n = \frac{\sum_{i=1}^{n} w_i}{n} \tag{9}$$

The values of the parameters $\lambda_3, \lambda_4$ could be estimated by a downhill simplex maximization of the correlation coefficient (Tarsitano, 2006, p. 19):

$$\rho(\lambda_3, \lambda_4) = \frac{\sum_{i=1}^{n} \left\{ \left[p_i^{-\lambda_4} - 1\right]^{-\lambda_3} - w_n(\lambda_3, \lambda_4) \right\} x_{i:n}}{\sqrt{S_x b_n(\lambda_3, \lambda_4)}}, \ i = 1, 2 \ldots n \tag{10}$$

where: $x_{i:n}; i = 1, 2 \ldots n$ is an ordered sample of wages of size $n$, $w_n = n^{-1} \sum_{i=1}^{n} w_{i:n}, i = 1, 2 \ldots n$;

$S_x = \sum_{i=1}^{n} (x_{i:n} - est\ \mu_n)^2$ and $b_n(\lambda_3, \lambda_4) = \sum_{i=1}^{n} (w_{i:n} - w_n)^2$.

New and already known relative inequality measures could be drawn analogically to formula for Kappa (9) with regard to its first factor in the formula (5), which depends on the shape of the probability distribution that generates incomes in population and could be defined by proper quantile function $Q(p) = p,\ 0 < p \leq 1.$

For example Gini index is measure $V_n$ to the uniform distribution with quantile fiction definition: $Q(p) = p,\ 0 < p \leq 1$. De Vergottini index corresponds $V_n$ with basic quantile form of exponencial distribution $Q(p) = -\ln(1-p),\ 0 < p \leq 1$. For Bonferroni index is the initial the basic quantile form of reflected-exponential distribution: $Q(p) = \ln p,\ 0 < p \leq 1$.

The Gini coefficient, De Vergottini rate, Bonferroni and Tarsitano measures of relative inequality, which could be defined by by Tarsitano proposed one generalized formula (5) with its value calculated by (7), thus should be analogous and ought to give comparable results in regional studies.

Definitions of Tarsitano's inequality measure $V_n$ based on rankits for quantile function $Q(p) = F^{-1}(p),\ 0 < p \leq 1$, with "the best fit" estimates of its shape parameters of a reasonably flexible kappa distribution allows to calculate inequality indices within-region inequality in regional comparisons.

## 2 Results

According Tarsitano theory the standard parametric approach to quantification and comparison of inequality in subsequent years based on income Kappa quantile models requires theirs appropriate and analytically identical definitions of model with actualisation of "good fit" parameters in each year.

Using the same methodology for geographical comparison, we evaluate within region inequality versus Kappa quantile models for wage distribution in each of the eight Slovak regions. To condense information on inequality of wages of employees of the Slovak Republic and its regions, all 9900 individual observations were retained in calculations of Tarsitano's inequality measures based on rankits in Q-Q plots using the formula (9). The values of both measures, the Tarsitano inequality measure *V*(inequal.) by the formula $V_n(\lambda_3, \lambda_4)$ in (9) and the correlation coefficient (corr. coef.) by the formula $\rho(\lambda_3, \lambda_4)$ in (10) are for NUTS III regions of the Slovak Republic displayed in Q-Q plots (Fig. 3) and are summarized in the Tab. 3 of the article Sipko and Sipková (2013). This article concluded that: "Based on the size of the acquired values of the unequal income distribution of employees in the various regions of the Slovak Republic, measured by Tarsitano's proposed measure (*V-inequality*), the order from the largest to the lowest values is as follows: Region 6, Region 7, Region 3, Region 4, Region 1, Region 8, Region 2 and Region 5. The order may be in our opinion interpreted in accordance with the theory by Amartya Sen (1997) as a sequence of "possible perception of inequality in the distribution of existing income distribution" of individual employees in the Slovak regions from the strongest to the weakest."

In addition we calculated the Gini, De Vergottini, and Bonferroni coefficients in terms of relative inequality versus basic quantile function of uniform, exponential, and reflected-exponential distribution in the geographical regions of the Slovak Republic. In calculation of $R_{i:n}$ in formula (7) was used point estimates of rankit expected values of order statistics based on $Q(p)$ with the same $p_i(\alpha, \beta)$ according to Landwehr ( $\alpha = 0{,}35; \beta = 0{,}65$ ):

$$V_n(\alpha, \beta; \boldsymbol{\lambda}) = \sum_{i=1}^{n} \left[ \frac{Q(p_i(\alpha, \beta); \boldsymbol{\lambda}) - w_n}{Q(p_n(\alpha, \beta); \boldsymbol{\lambda}) - w_n} \right] \left( \frac{X_{i:n}}{T} \right); \tag{11}$$

where: $T = n\mu_n = \sum_{i=1}^{n} X_{i:n}$ ; $w_n = n^{-1} \sum_{i=1}^{n} Q(p_i(\alpha, \beta); \boldsymbol{\lambda})$; $w_{i:n} = Q[p_i(\alpha, \beta)]$

$$p_i(\alpha, \beta) = \frac{i - \alpha}{n + 1 - (\alpha + \beta)} = (i - 0{,}375)/n, \quad i = 1, 2 \ldots n, \tag{12}$$

But the resulted ordering of regions by level of $V_n$ - regional inequality measures with Kappa $Q(p)$ does not correspond to Gini, De Vergottini and Bonferroni measures calculated by the same generalized formula (5). Results are summarized in Tab. 1.

The Gini, De Vergottini and Bonferroni measures, however, did not contradict each other. Rank of Slovak regions was under them identical. In line with the known regional expert analyses Bratislava region (Region 1) has the highest inequality, what the three measures confirmed. Only empirical comparison of shapes to the estimated Kappa models for each region has proved to be problematic. None of the three measures compare the empirical distribution shape to the theoretical model, which parameters would be necessary to estimate for each region separately. Shape of uniform distribution, the basic shapes of exponential distribution and reflected-exponential distribution only depend on the position $p_i(\alpha, \beta)$.

In the next step, unified theoretical shape Kappa of the overall distribution of the SR was applied for calculation of the Tarsitano SR regional relative inequality measures and compared for eight regions graphically with the three relative inequality measures (Fig. 1).

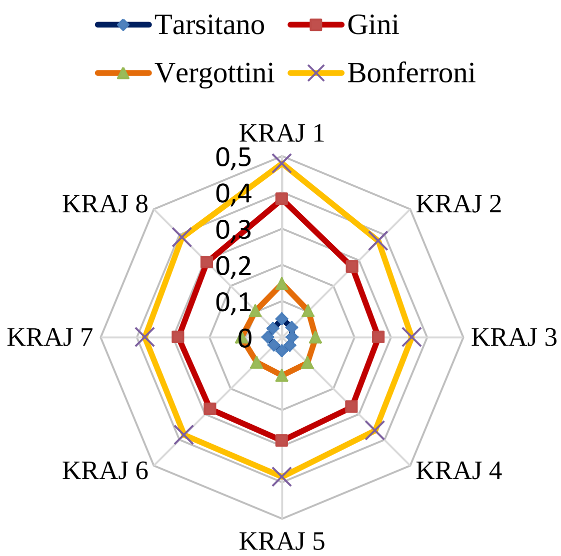
**Tab. 1: Values of Tarsitano, Gini, De Vergottini and Bonferroni inequality measures of wages of employees of the Slovak Republic and its regions**

| Region SR | corr. coef. | Tarsitano | Tarsitano SR* | Gini | Vergottini | Bonferroni |
|---|---|---|---|---|---|---|
| SR | 0.9920 | 0.0210 | 0.0210 | 0.3210 | 0.0973 | 0.4175 |
| 1 | 0.9832 | 0.0466 | 0.0501 | 0.3808 | 0.1480 | 0.4795 |
| 2 | 0.9792 | 0.0409 | 0.0383 | 0.2741 | 0.1027 | 0.3756 |
| 3 | 0.9278 | 0.0499 | 0.0281 | 0.2663 | 0.0930 | 0.3577 |
| 4 | 0.9597 | 0.0468 | 0.0316 | 0.2719 | 0.0996 | 0.3634 |
| 5 | 0.9896 | 0.0401 | 0.0381 | 0.2855 | 0.1054 | 0.3848 |
| 6 | 0.9301 | 0.0546 | 0.0306 | 0.2799 | 0.0979 | 0.3812 |
| 7 | 0.9562 | 0.0501 | 0.0390 | 0.2856 | 0.1114 | 0.3776 |
| 8 | 0.9826 | 0.0420 | 0.0343 | 0.2924 | 0.1030 | 0.3893 |

* Parameters of fitted Kappa model SR are $\lambda_3 = 0{,}444230372$ and $\lambda_4 = 0{,}819436133$

Source: Own analysis by Information System of Labour Costs (ISCP) data, 2010.

**Fig. 1: Comparison of Tarsitano SR, Gini, De Vergottini and Bonferroni measures of wages of employees of the eight regions of Slovak Republic**



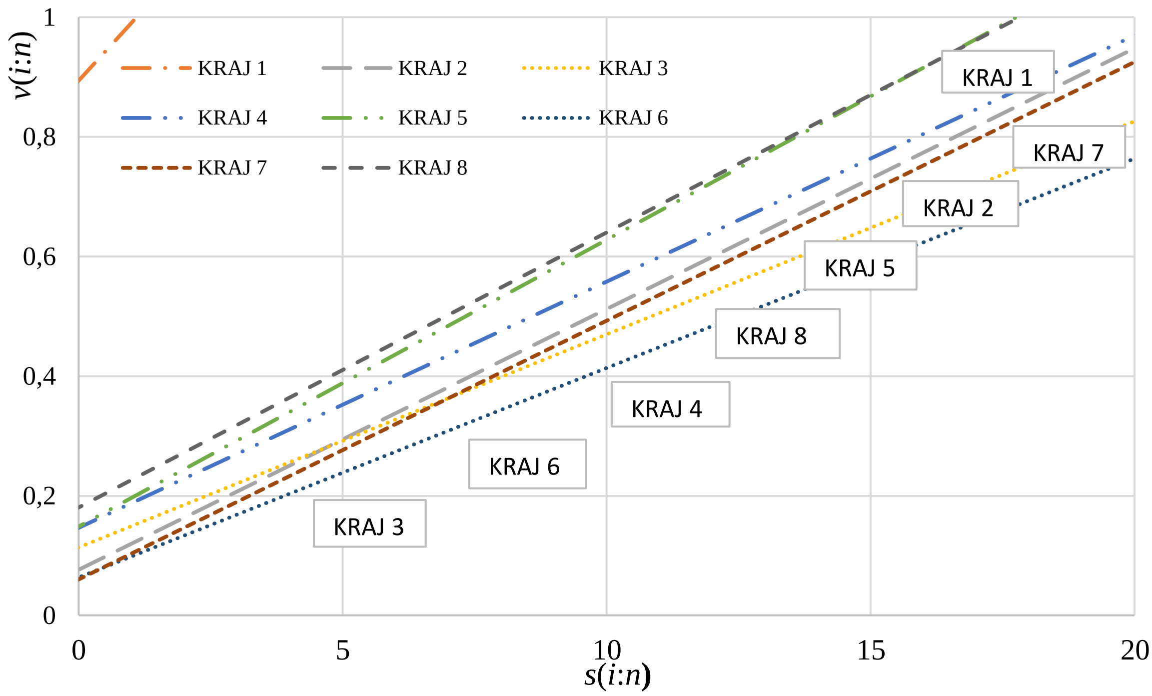

Source: Own analysis by Information System of Labour Costs (ISCP) data, 2010.

Each coefficient of inequality from the above measures the same concept, is calculated by the same formula (7) and the order of regions is the same, only results are in a different scale of measurement. This is consistent with the empirical values of deviations from the estimated "expected values of fair-equal" income distribution.

The lowest values (degree of deviation from the kappa model SR) are in the case of Tarsitano measures, followed by the size of De Vergottini rates (for deviations from the exponential distribution), then the Gini coefficients (uniform distribution) and the largest values have Bonferroni rates (reflected-exponential distribution).

In the new regional comparison of relative inequality under Tarsitano generalized inequality measure with identical basis of the "expected fair-equal" overall income distribution of the SR, we propose graphical presentation in one *XY*-chart according to the values $s_{(i:n)} = \left(x_{i:n} - est\,\mu_n\right)^2$ on the $x$ axis and the values $v_{(i:n)}$ on $y$-axis calculated according to equation (5). The resulting order of the eight regions of the SR by their additions to overall relative inequality measure for the SR we display by the slopes, which are never intersect as Lorentz curves often do (Fig. 2).

**Fig. 2: Geographical relative wage inequality composition of Tarsitano $V_n$ measure of the SR by the slopes of *Q-Q* regional graphs (Kappa income model).**




Source: Own analysis by Information System of Labour Costs (ISCP) data, 2010.

## Conclusion

Comparison of relative inequality in the regions of the SR with application of the overall Kappa model for the SR leads to the same order of regions according to the Gini, DeVergottini and Bonferroni measures of inequality, which could be defined by one generalized formula proposed by Tarsitano using different types of quantile models. Different is the only scale of the four measures of inequality. Furthermore, by calculation of Tarsitano's inequality measures in regional studies individually for each region based on rankit of one choosen quantile income model of "fair equal income distribution in the country" is possible to condense information on total within-regions relative income inequality in one measure and displayed it in one Q-Q plot with the corresponding contributions of each region.

## Acknowledgment

Research was partially supported and funded by the VEGA Project No.01/0127/11: The spatial distribution of poverty in the European Union.

## References

DAVID, H.A.; NAGARAJA, H. N. (2003). *Order Statistics*. 3rd Edn. John Wiley and Sons, USA. ISBN 0-471-38926-9.

Gilchrist, W.G. (2000). *Statistical modelling with quantile functions*. Chapman & Hall. ISBN 1-5848-8174-7.

HOSKING, J. R. M. (1994). The four-parameter kappa distribution. *IBM Journal of Research, Development*, Vol. 38, No. 3, 251-258.

LOUNGANI, P. (2003, September). Inequality : Now you see it, now you don't. *Finance & Development*. Publisher: IMF & INT BANK OF RECONSTRUCTION AND DEVELOPMENT, Vol. 40, No. 123, 22 – 23.

RAVALLION, M. (2003). The debate on globalization, poverty and inequality: Why measurement matters. *International Affairs*, Vol. 79, No. 3, 739-753.

Sen, A. K. (1997). *On economic inequality*. Oxford : Clarendon Press.

SIPKO, J., SIPKOVÁ, Ľ. (2013). Income inequality in the Slovak Republic. *Research journal of economics, business and ICT*. London : English Time Schools & Overseas Education, Vol. 8, No. 1, 5-11. ISSN 2045-3345.

SIPKOVÁ, Ľ., SIPKO, J. (2012). Analysis of income inequality of employees in the Slovak Republic. *The 6th international days of statistics and economics : conference proceedings*, September 13-15, 2012, Prague, Czech Republic. Prague : University of Economics Prague. ISBN 978-80-86175-86-7. 11 p., online.

SIPKOVÁ, Ľ., SODOMOVÁ, E. (2007). *Modelovanie Kvantilovými funkciami*. Bratislava : Ekonóm. ISBN 978-80-225-2346-2.

STANKOVIČOVÁ, I. (2010). Regionálne aspekty monetárnej chudoby na Slovensku. In: Pauhofová, I., Hudec, O., Želinský, T. (eds.): *Sociálny kapitál, ľudský kapitál a chudoba v regiónoch Slovenska*. Košice: TU Košice. 67-75. ISBN 978-80-553-0573-8.

TARSITANO, A. (2006). A new Q-Q plot and its application to income data. *Statistica & Applicazioni*. Vol. IV, special issue No. 1.

VOJTKOVÁ, M., LABUDOVÁ, V. (2010). Regionálna analýza príjmov a výdavkov v Slovenskej republike. *Ekonomický časopis/Journal of Economics*. Vol. 58, No. 8, 802-820.

## Contact

Ľubica Sipková
University of Economics, Faculty of Economic Informatics
Dolnozemská cesta 1, 852 35 Bratislava, Slovak Republic
lubica.sipkova@euba.sk

Juraj Sipko
Paneuropean University, Faculty of Economics and Business
Tematínska 10, 851 05 Bratislava, Slovak Republic
jurajsipko@gmail.com