AN APPLICATION OF CLUSTER ANALYSIS ON THE
POLISH HOUSING MARKET

Patrycja Kowalczyk –Rólczyńska

Abstract
The real estate market is one of the most important markets in the economic development of the country. This market is strongly related to the mortgage market, the labor market and the building sector. Analysis of the real estate market is related to the assessment of the development of the many factors which describe this market. The factors describing the real estate market are: prices on the real estate market, real estate entities, investments in this market, transactions on this market and the features of the real estate. It follows that the analysis of the real estate market should be multidimensional.

The purpose of this article is application of cluster analysis to housing market in selected cities in Poland. In the article there was used one of hierarchical methods of multi-feature object clustering – the Ward’s method. The analysis included housing markets in sixteen biggest Polish cities in the period 2009-2012.

Key words: housing market, cluster analysis, factors influencing housing market

JEL Code: R310, C180, C380

Introduction
The housing market plays an extremely significant role in the economy. It is under the influence of and it exerts influence on the development of the economy among others through generating gross domestic product, supplying the necessary space to satisfy housing needs, engaging a considerable number of employees, participating in taxes (local, on income generated by real estate, on sales, on the appreciation of real estate), developing financial products needed for real estate purchase and insurance. Given also the high number of factors describing the housing market and factors contributing the development of this market, it has become the object of numerous studies (see e.g. Fischer & Stamos 2013; Renaud et al. 2001; Genesove & Mayer, 2001).
An analysis of the Polish housing market is conducted in this paper with the use of one of hierarchical methods of multi-feature object clustering – the Ward’s method. The analysis included housing markets in sixteen biggest Polish cities\(^1\) in the period 2009-2012. Due to the fact that real estate markets, and housing markets in particular, are local in nature (which arises from the association of real estate with land), the cities were clustered by adopted criteria to enable the capturing of convergences and identical trends occurring on local voivodship markets.

1. Factors describing the housing market

The Polish housing sector, although it is often analysed as a whole, is a heterogeneous market that is largely diversified across the individual 16 voivodship capital cities. In order to cluster local real estate markets and conduct an analysis of the identical trends occurring on these markets, factors describing each of the local housing markets were chosen. These factors include (with reference to each analysed year and each analysed city)\(^2\):

- Transaction prices on the primary housing market,
- Transaction prices on the secondary housing market,
- Total number of apartments,
- Number of delivered apartments,
- Number of issued residential building permits,
- Average usable floor space in an apartment,
- Average usable floor space in a delivered apartment,
- Average number of rooms in an apartment,
- Average number of rooms in a delivered apartment,
- Number of entities operating on the real estate market (including developers, real estate agents, real estate surveyors, housing cooperatives),
- Number of transactions concluded on the housing market,
- Value of the transactions concluded on the housing market.

Transaction prices of residential real estate on the primary housing market depend on the location. It is observed that the majority of projects in cities are implemented in their suburbs, outside the city centre. This arises mainly from the availability and price of land on such areas. Where the price of land is high and the location is good (near or in the city centre),

\(^1\) Poland is divided into 16 voivodships. The object of the analysis is housing markets in each capital city of a given voivodship.

\(^2\) These are factors used for the purpose of analysing local housing markets carried out by the National Bank of Poland (see: NBP, 2013)
apartment prices are extremely high, which results in low interest in them. A large number of persons purchasing apartments use a mortgage loan and not many households can obtain a high loan amount (exceeding PLN 500,000). It needs to be emphasised that the secondary housing market offers cheaper apartments than the primary one. This regards mainly apartments situated in big housing estates constructed with the so-called large panels or in the post-war period, the wear of which is considerable. However, location is an asset of such real estate.

The development of the housing market is proved by the number of delivered apartments and the number of issued building permits. The increasing number of delivered apartments and the growing number of issued building permits proves the continuous housing needs on a given local market. Moreover, this means that the enterprises constructing new residential real estate achieve the expected rates of return on investment and willingly implement new projects (since they are profitable).

The average usable floor space in an apartment and the average number of rooms in an apartment are often adapted to the demand on the local housing market. Moreover, the usable floor space is limited in Poland by government programmes aimed to support the housing development industry. The programmes have been implemented for several years and are dedicated to young people (i.e. ones not older than 35 years). In both significant government programmes, i.e. “Family in its own home” (“Rodzina na swoim”) and “A home for the young” (“Mieszkanie dla Młodych”) the maximum apartment floor space is 75 sq m and 85 sq m (if the apartment purchaser raises at least three children), respectively.

An important position on the real estate market is held by the entities that operate thereon. These include developers, real estate agents, real estate surveyors, and housing cooperatives. The increasing number of such entities contributes to competition on the real estate market and to lower costs of the services provided thereby (commission fees for agents, charges for preparing the property appraisal report) and to lower apartment prices.

The transactions concluded on the housing market (both with reference to the number and value of such transactions) imply the local nature and the development of the market. The low number of concluded transactions might be a reflection of a too low demand or a demand that fails to satisfy needs and expectations of apartment purchasers. The growing number of

---

3 In 2013, only 5.63% of borrowers obtained mortgage loans of over PLN 500,000 (i.e. circa EUR 119,000)
4 Submitting applications for funding as part of this programme has already completed while the programme was intended to apply for 8 years and therefore the state support is still being granted.
concluded transactions and the increasing number of delivered apartments might prove that the supply on this real estate segment corresponds to the demand.

2. Research methods – the Ward’s method
Hierarchical methods are classification methods used for the purpose of clustering by similarities and dividing by differences of the objects subject to examination. Classification methods and related problems are quite specifically discussed in the relevant literature (see e.g. Jardine & Sibson, 1968; Ward, 1963; Rand, 1971; Ball, 1965).

The Ward’s method is one of agglomerative hierarchical methods. Hierarchical methods of multi-feature object clustering, proposed by G. M. Lance and W. T. Williams (1967), are characterised by several variants differing by the manner of designating the matrix of distance between object clusters in a multidimensional classification space. The Ward’s method is a specific case of a general object clustering scheme developed by G. M. Lance and W. T. Williams.

The pattern of conduct is the same for all agglomerative methods and can be called the central agglomerative procedure. It is assumed that the point of departure is matrix D of Euclidean distances \( d_{ij} \) between the classified objects \( O_1, O_2, \ldots, O_N \).

The central agglomerative procedure allowing for the Ward’s method is as follows (Nowak, 1990):

1. Each object \( O_i \) (\( i = 1,2,\ldots,N \)) is treated as a single-element cluster,
2. The minimum value \( d_{pq} = \min_{i,j} d_{ij} \) is sought in the distance matrix;
3. Objects \( O_p \) and \( O_q \) are treated as single-element clusters, \( A_p \) and \( A_q \) are merged in one two-element cluster \( A_r : \)
   \[ A_r = A_p \cup A_q ; \]
4. Next the distance \( d_{ir} \) of the newly created cluster \( A_r \) from all other clusters \( A_i \) is designated in accordance with the selected method. Distance \( d_{ir} \) is put to matrix \( D \) in the place of the p-th line and p-th column, eliminating at the same time q line and column;
5. Steps 2–4 are repeated until all objects create one cluster.
After each iteration of the agglomerative hierarchical procedure a division of comparable objects into an increasingly smaller number of clusters is obtained. The outcome is the current matrix of distance between clusters.

The general formula for transforming the distance matrix while merging clusters \( A_p \) and \( A_q \) to form a new cluster \( A_r \) for hierarchical agglomerative methods operating in accordance with the principle of central agglomerative procedure is as follows:

\[
d_{ir} = a_p d_{ip} + a_q d_{iq} + b d_{pq} + c \sqrt{d_{ip} - d_{iq}}
\]  

(1)

where:

\( d_{ip} \) – distance between clusters \( A_i \) and \( A_p \),
\( d_{iq} \) – distance between clusters \( A_i \) and \( A_q \),
\( d_{pq} \) – distance between clusters \( A_p \) and \( A_q \),

Values \( a_p \), \( a_q \), \( b \) and \( c \) are distance matrix transformation parameters. For the Ward’s method they are designated based on the following formulas:

\[
a_p = \frac{N_i + N_p}{N_i + N_r}
\]  

(2)

\[
a_q = \frac{N_i + N_q}{N_i + N_r}
\]  

(3)

\[
b = -\frac{N_i}{N_i + N_r}
\]  

(4)

\( c = 0 \).

Symbols \( N_i \), \( N_p \), \( N_q \) and \( N_r \) denote the number of elements in clusters \( A_i \), \( A_p \), \( A_q \) and \( A_r \).

A dendrogram, which is a graphic interpretation of the outcome, is obtained from the analysis.

The Ward’s method intends to minimise the sum of squared deviations of any two clusters, which can be formed at each stage. It is believed that the Ward’s method is a very effective method and it intends to obtain small clusters (Grabiński & Sokołowski, 1984).

In order to use the above-described method, appropriate diagnostic features need to be selected. The selection of such features can be considered an extra-statistical or a statistical task. In the former case, the diagnostic features are the values that most accurately illustrate the features of the compared objects in light of the acquired substantive knowledge about the
examined phenomenon. In the latter case, the selection of appropriate diagnostic features is determined by selected statistical methods. One of the statistical criteria for selecting features is variation of a given diagnostic feature which is determined for the purpose of checking whether a given diagnostic feature is capable of discriminating the examined objects or is characterised by sufficiently high variation. The selection of features with the use of this criterion consists in eliminating the features whose variation coefficients
\[ V_i = \frac{s_i}{\bar{x}_i} \]
where \( s_i \) is standard deviation of feature, \( \bar{x}_i \) is average value of feature \( x_i \) are lower in terms of the module than the assumed critical value \( V^* \) from a given set of potential diagnostic features since such features are believed to be characterised by too low variation.

### 3. Results of the research

The research included 16 local housing markets in Poland in the period 2009-2012. Potential diagnostic features were created based on the factors described in section 1, which were selected substantively as features describing the housing market. Additionally, variability coefficients of potential diagnostic features were calculated in order to check whether they are capable of discriminating the examined housing markets. The values of the coefficients correspond to the 2012 data since they are the most up-to-date. It is worth mentioning that variability coefficients of diagnostic features were not calculated for each year separately as it could turn out that different diagnostic features would be characterised by too low variability each year and they should be removed from the set of potential diagnostic features. This would mean that the set of diagnostic features would be different for each examined year. In such a situation, a comparison of local housing markets over the examined period, that is four years, would not be possible. The values of variability coefficients for the largest cities on the Polish housing market are presented in Table 1.

#### Tab. 1: Variability coefficients of diagnostic features for 2012

<table>
<thead>
<tr>
<th>Name of a diagnostic feature</th>
<th>Variability coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction prices on the primary housing market</td>
<td>0.155</td>
</tr>
<tr>
<td>Transaction prices on the secondary housing market</td>
<td>0.261</td>
</tr>
</tbody>
</table>

5 Variation coefficient is a measure of relative dispersion of a data series (Makridakis et al., 1998)

6 The analysis included real estate markets in the following Polish cities: Białystok, Bydgoszcz, Katowice, Kielce, Kraków, Lublin, Łódź, Olsztyn, Opole, Poznań, Rzeszów, Szczecin, Gdańsk, Warsaw, Wrocław, Zielona Góra.
The obtained values of variability coefficients prove that the features describing the number of entities operating on the real estate market and the transactions concluded on the housing market are characterised by the highest variability. The lowest variability, in turn, is inherent in the features related to the average number of rooms in an apartment, the average usable floor space in an apartment, and the number of apartments per resident.

Due to the fact that the critical value of the variability coefficient was assumed to be $|\hat{V}^*| = 0.05$, all of the above potential diagnostic features were included in the set of diagnostic features. Then, local housing markets were clustered with the use of the Ward’s method\(^7\), and the results of the research are presented on the dendrograms below.

---

\(^7\) The computer program Statistica was used for the calculations.
The conducted clustering of local housing markets allowed the separation of four clusters in the period 2009-2010 and three clusters in the period 2011-2012. The table below includes the classification results.

**Tab. 2: Results of clustering local housing markets with the use of the Ward’s method in the period 2009-2012**

<table>
<thead>
<tr>
<th>Class</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Łódź, Katowice,</td>
<td>Łódź, Katowice,</td>
<td>Łódź, Katowice,</td>
<td>Łódź, Katowice,</td>
</tr>
<tr>
<td></td>
<td>Olsztyn, Lublin,</td>
<td>Olsztyn, Lublin,</td>
<td>Olsztyn, Lublin,</td>
<td>Olsztyn, Lublin,</td>
</tr>
<tr>
<td></td>
<td>Szczecin, Bydgoszcz,</td>
<td>Szczecin, Bydgoszcz,</td>
<td>Szczecin, Opole,</td>
<td>Szczecin, Opole,</td>
</tr>
</tbody>
</table>

**Source:** own work with the use of Statistica program
The results of clustering unambiguously prove that the housing market in Warsaw, which is the capital city of Poland, stood out in the analysed years. It needs to be stressed that housing markets in Poznań, Gdańsk and Wrocław are similar to each other (since they are always in the same class), like in the case of Opole and Zielona Góra housing markets. Moreover, Łódź, Katowice, Olsztyn, Szczecin, Bydgoszcz, Kielce, Białystok housing markets are also similar markets.

**Conclusion**

The research carried out above permitted the classification of local housing markets in Poland and identification of similar markets. Diagnostic features were the ones describing the housing market that are used in the research conducted by the National Bank of Poland. The results of clustering local housing markets with the use of the Ward’s method in the period 2009-2012 prove that the housing market in the capital city of Poland is a market that stands out from the rest (it was only in 2012 that the Kraków housing market was similar). It needs to be emphasised that changes occur on local housing markets in Poland, which is proved by shifts of certain markets between classes.

**References**


**Contact**

Patrycja Kowalczyk-Rólczyńska

Wrocław University of Economics, Department of Insurance

Komandorska 118/120

53-345 Wrocław, Poland

Mail: patrycja.kowalczyk@ue.wroc.pl