EQUIVALENCE SCALES FOR EVALUATION OF EQUIVALISED INCOME OF CZECH HOUSEHOLDS

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Abstract

In the modelling of incomes the equivalised incomes based on incomes of households are frequently used in order to make adjustments to the actual incomes of households in a way that enables analysis of the relative wellbeing of households of different size and composition (number of adults, number of children and their age). In the contribution, different methods of evaluation of equivalised units (equivalised number of members) are discussed and the pros and cons are shown for data dealing with Czech households´ incomes from 2005. Equivalised household income can be viewed as an indicator of the economic resources available to each individual in a household, but unfortunately no generally accepted and optimal methodology exits. The strong dependence is expected and quantified (from different point of view) between all treated equivalised incomes. Moreover the distributions of equivalised incomes in different types of households are modelled (and compared) with the use of lognormal and Dagum distributions. Maximum likelihood estimates of parameters are found with R program.

Key words: income distribution, equivalised units, equivalised income, household

JEL Code: D31, C23, D12

Introduction

The expenditures of a household grow with each additional member but not in proportional way because of the possibility of sharing needs for housing, electricity, heating etc. The equivalence scales try to assign to each household a value in proportion to its needs. These values (also called equivalent units, equivalent adults) enables to evaluate equivalised incomes representing an income for a standardize household or an income by one equivalised adult. The choice of an equivalence scale depends on technical assumptions about economies of scale in consumption as well as on value judgments about the priority assigned to the needs of different individuals (in case of adults and children). Equivalised household income is an indicator of the economic resources available to each member of the household (OECD, 2015). It can be used for the analysis of incomes and for comparison of incomes in different countries or regions as well as for analysing poverty or danger of poverty (Buhmann et all. (1988); Flachaire and Nunez (2007); Deaton (1997)).

The aim of this contribution is to compare four equivalised incomes (per capita and with three definitions of equivalent units defined by modified methodology of Organisation for Economic Cooperation and Development (OECD-modified scale), methodology of OECD (OECD-scale) and a square root scale). The impact of the choice of equivalence scale on equivalised income will be of interest from different points of view. The probability distributions of equivalised incomes in the Czech Republic in 2005-2010 are treated and compared in the text. Two income distributions (lognormal and Dagum) are fitted into data from the survey „Životní podmínky“ 2006-2011 conducted regularly by the Czech Statistical Office. The development of parameters is shown and estimated characteristics of the level and variability are given and compared.

1 Equivalised incomes

In this part various definitions of equivalent scales are introduced (OECD, 2015; Jorgenson and Slesnick (1987)). The easier approach is to use number of members

$$sc = \text{number of members.} \qquad (1)$$

In order to reflect stronger possibility to share spendings, square root of number of members could be used

$$rootsc = \sqrt{\text{number of members}}. \qquad (2)$$

For a household with four members we obtain $sc = 4$ a $rootsc = 2$ and for all households it holds $sc \geq rootsc$. It means that a household of 4 person has needs twice as large as a single member household.

The methodology of the OECD (OECD-scale, denoted by *sj*) assign to members of the household weights

first adult: 1.0 ; other adults above 13: 0.7; child below 13: 0.5. (3)

Modified OECD scale (modified OECD-scale, denoted *msj*) takes more into account sharing of expenditures and the weights are defined as

first adult: 1.0; other members above 13: 0.5; child below 13: 0.3. (3)

All these definitions evaluate 1 for single member household and to the household of two adults 2 (*sc*), $\sqrt{2} = 1.41$ (*rootsc*), 1.7 (*sj*) or 1.5 (*msj*). The Jensen´s equivalence scale (Jensen in 1988 for Australia and New Zealand) is given as

$$\frac{(a + xc + yt)^z}{2^z},$$

where *a* is number of adults, *c* number of children, *t* total age of all children and *x, y, z* are constants. For a household of two adults $(c = t = 0)$ in this case we obtain (regardless constants *x*, *y*, *z*) number of units 1 and for single member family 0.5^z.

According to the (1), (3)-(4) we obtain

$$sc \geq sj \geq msj, \qquad (5)$$

and equality is reached for single member households.

We will define equivalised annual net income of a household as a ratio of an annual total net income (CZK) and number of equivalised units.

We will fit three parameter lognormal and Dagum distributions, these distributions are supposed to be good model for income or wages (Kleiber and Kotz, 2003; Bílková, 2012) for multivariate lognormal distribution see Bartošová and Longford (2014). The three parameter Dagum distribution, called also Inverse Burr´s, is defined by the density

$$f_{Dag}(x;\alpha,\beta,p) = \frac{\alpha p y^{\alpha p - 1}}{\left(\beta^{\alpha p}(1 + (x/\beta)^{\alpha}\right)^{p+1}}, x > 0, \qquad (6)$$

where α, β and *p* are positive parameters, lognormal distribution with parameters $\mu \in R, \sigma^2 > 0$ and $\theta \in R$ is described by the density

$$f_{LN}(x;\mu,\sigma^2,\theta) = \frac{1}{\sqrt{2\pi}\sigma x}\exp\left(\frac{(\ln x - \mu)^2}{2\sigma^2}\right), x > \theta.$$

There exist explicit formulas for different characteristics of both distributions (Kleiber and Kotz, 2003), the parametrizations coincide with these distribution in R packages. These distributions are not supposed to fit highly non-homogenous distribution of incomes in the set of all households, but it can be accepted as a suitable model for comparison of different equivalised incomes. Unknown parameters are estimated by maximum likelihood method (modified in case of lognormal distribution) in R with the use of packages Fitdistrplus

(Delignette-Muller, Dutang, 2015) and VGAM (Yee, 2010). It is known (Bílková, 2012), that maximum likelihood estimates are sensitive to isolated observations (and there are isolated large incomes in our data) but the sample sizes are large enough to obtain reasonably good fits. Models are compared by the value of logarithmic likelihood in solution because both distributions are treated as three parametric.

2. Data and Results

In this part of the article we will use data from Living Conditions Survey (LCS, in Czech "Životní podmínky" within the project European Union – Statistics on Income and Living Conditions) from six consecutive years 2006-2011. The survey has been held by the Czech Statistical Office yearly since 2005, the survey LCS 2006 refers to the incomes from 2005 etc., so the analysed period covers incomes from years 2005 to 2010. The aim of the survey is to gather representative data on income distribution for the whole population and for various household types. For each household in the sample a net annual total income (in CZK) was divided by number of equivalent units *msj, sj, sc* and *rootsc* in order to evaluate equivalised incomes (adult equivalent income) *CPMSJ, CPSJ, CPPC* and *CPSO*.

In the Table 1 estimated parameters are shown for Dagum distribution and all equivalised incomes, for lognormal distribution estimated parameters are given in Table 2. The development of estimated parameters in time is well visible. Parameters β are increasing in time, while other parameters are oscillating. Estimates of the parameter *p* are similar for *CPMSJ* and *CPSO*, for *CPSJ* it is smaller and the smallest value of estimated *p* is for the smallest *CPPC* (with the highest value of estimated α).

Table 1: Maximum likelihood estimates of unknown parameters (Dagum distribution)

income	*msj*			*sj*			*sc*			*rootsc*		
year	$\hat{\alpha}$	$\hat{\beta}$	$\hat{p}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{p}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{p}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{p}$
2005	3.822	114 856	1.609	4.069	106 085	1.445	4.153	99 382	1.083	3.583	121 826	1.595
2006	3.898	126 231	1.560	4.152	115 429	1.437	4.246	107 290	1.097	3.653	134 669	1.532
2007	4.025	139 831	1.477	4.276	125 147	1.441	4.327	113 773	1.163	3.746	148 977	1.462
2008	3.979	149 761	1.513	4.234	133 808	1.499	4.283	121 146	1.234	3.717	160 159	1.477
2009	4.012	160 134	1.398	4.255	144 834	1.325	4.306	131 828	1.078	3.774	170 630	1.387
2010	3.916	161 181	1.422	4.122	145 601	1.360	4.306	131 828	1.078	3.719	172 629	1.389

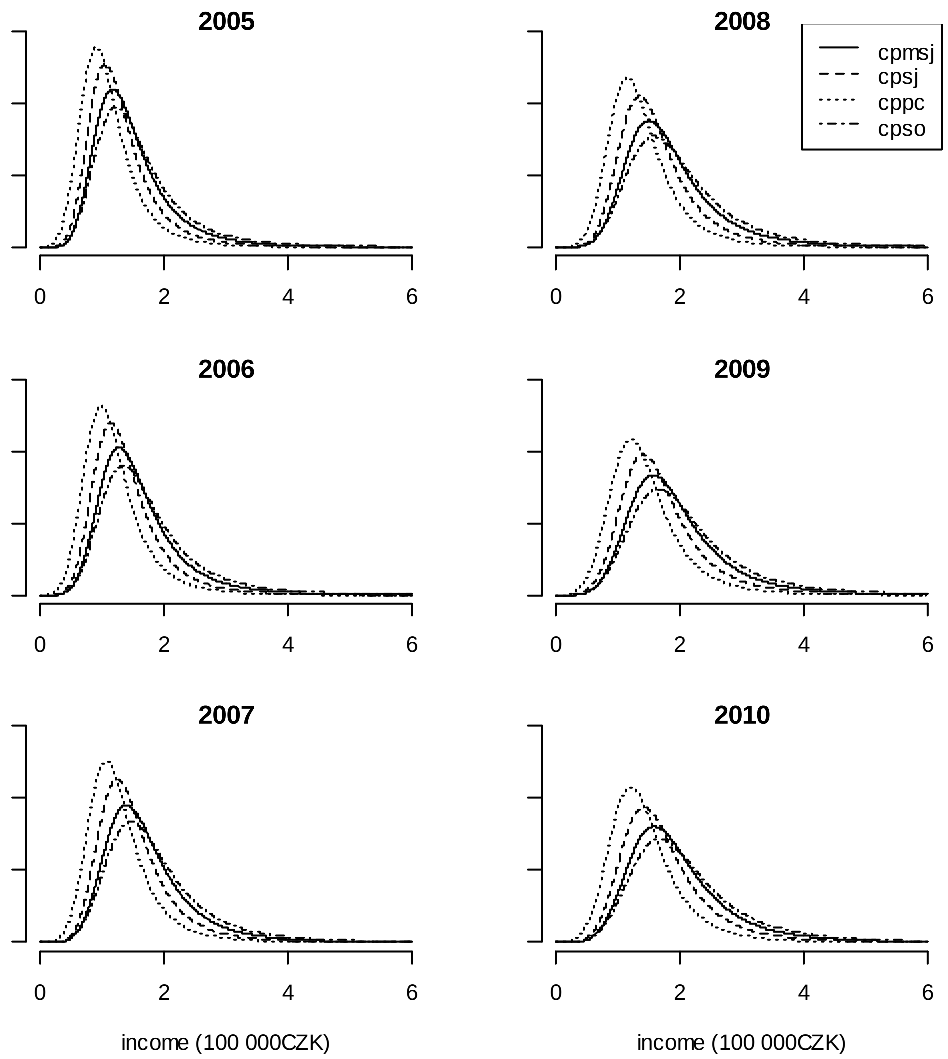
Source: own computations

Table 2: Maximum likelihood estimates of unknown parameters (lognormal distribution)

income	*msj*			*sj*			*sc*			*rootsc*		
year	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\theta}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\theta}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\theta}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\theta}$
2005	11.832	0.428	1000.01	11.705	0.421	1000.04	11.531	0.451	1000.02	11.899	0.450	1000.01
2006	11.912	0.422	1000.00	11.786	0.414	1000.00	11.613	0.440	999.97	11.981	0.445	1000.00
2007	11.991	0.417	999.97	11.863	0.406	1199.93	11.693	0.426	999.92	12.060	0.441	999.97
2008	12.070	0.420	999.99	11.945	0.407	1199.99	11.778	0.424	999.98	12.138	0.443	999.99
2009	12.109	0.426	999.98	11.982	0.417	1199.98	11.814	0.438	999.96	12.176	0.446	999.98
2010	12.124	0.429	1000.00	12.000	0.421	1200.00	11.814	0.438	999.96	12.190	0.447	1000.00

Source: own computations

Fig. 1: Estimated densities for Dagum distribution

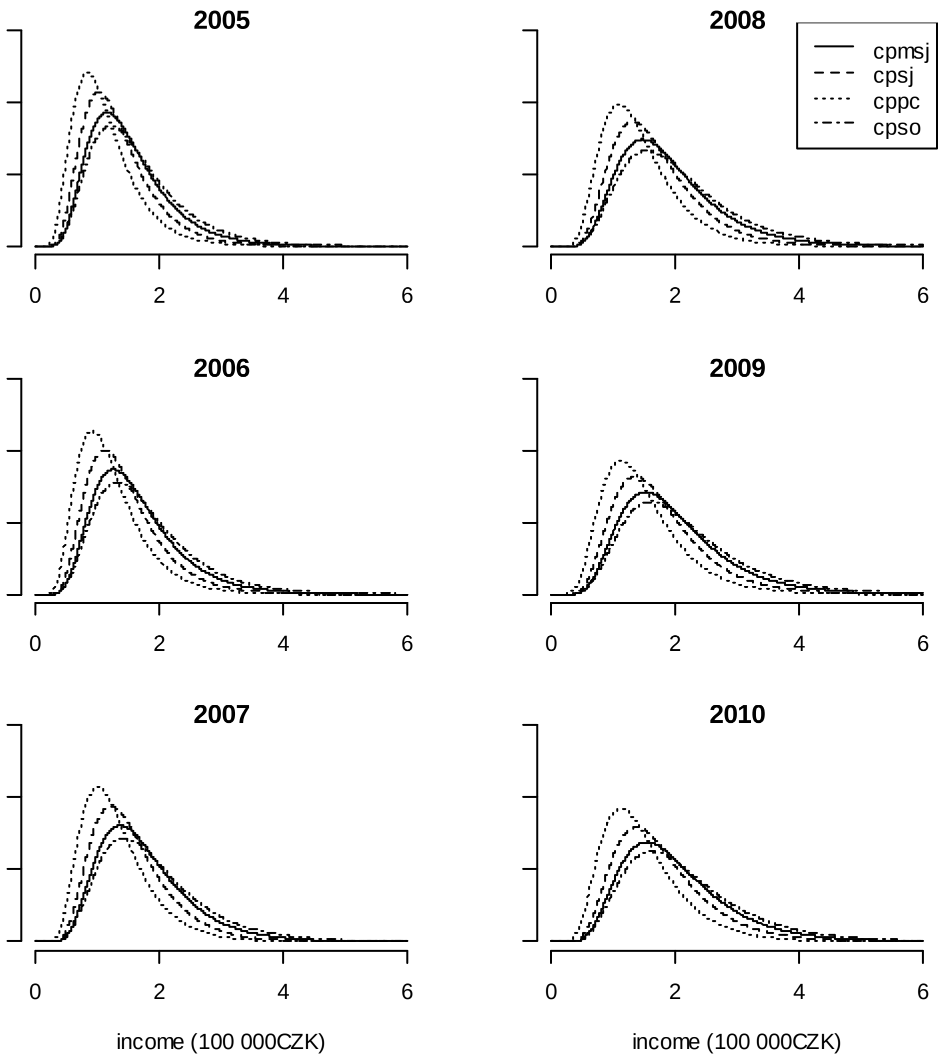


Source: own computations

For lognormal distribution all estimates of the shift parameter θ are similar (and fortunately positive). Estimated parameters μ are increasing while scale parameters show no trend. The impact of parameters and their development is well visible in the Table 3.

Estimated densities for all analysed years and equivalised incomes are shown in the Figure 1 (Dagum distribution) and Figure 2 (lognormal distribution). All figures have the same scales on both axes. All distributions are positively skewed (with coefficient of skewness approximately 3).

Fig. 2: Estimated densities for lognormal distribution



Source: own computations

All equivalised income are (for given year) highly correlated. It is obvious from the definition, because all incomes are ratios with the same nominator and similar denominators. Correlation coefficients between logarithms of equivalised incomes are (for all years) in the limits 0.8-0.97. Higher coefficients are for OECD-scales and for incomes based on number of household members.

Tab. 3: Sample and estimated characteristics of the level (median) and variability (quartile deviation) (in CZK)

income		sample		Dagum		lognormal	
	year	*median*	*q*	*median*	*q*	*median*	*q*
msj	2005	132 613	34 774	135 050	35 071	137 558	40 246
	2006	143 548	37 563	146 520	37 516	149 116	43 042
	2007	156 267	40 165	158 822	39 783	161 268	45 994
	2008	169 120	43 331	171 665	43 299	174 630	50 182
	2009	176 273	44 927	178 848	45 493	181 445	52 831
	2010	178 969	46 176	181 501	47 147	184 274	54 103
sj	2005	116 544	27 340	119 520	29 745	121 168	34 886
	2006	126 000	29 734	129 522	31 608	131 352	37 128
	2007	136 035	31 398	140 079	33 150	141 886	39 318
	2008	147 659	34 498	151 686	35 962	154 050	42 812
	2009	155 044	35 866	158 186	38 341	159 907	45 535
	2010	156 706	37 597	160 755	40 014	162 687	46 850
pc	2005	100 640	24 000	102 037	26 686	101 871	31 449
	2006	108 744	22 501	110 527	28 163	110 513	33 284
	2007	117 497	25 888	119 275	29 349	119 692	34 879
	2008	126 596	28 379	129 407	31 691	130 392	37 825
	2009	132 794	30 045	135 036	34 077	135 143	40 539
	2010	134 815	31 874	137 475	35 646	137 861	41 906
so	2005	142 548	40 851	144 370	40 149	147 100	45 297
	2006	154 406	44 897	156 917	43 110	159 653	48 593
	2007	168 017	47 845	170 242	46 020	172 846	52 181
	2008	181 908	51 722	183 839	49 988	186 854	56 728
	2009	188 795	52 955	191 382	51 925	194 088	59 212
	2010	191 376	53 501	194 080	53 449	196 808	60 280

Source: own computations

In the Table 3 estimated characteristics of location (median) and variability (quartile deviation) are given (for all incomes, years and both fitted distributions) and compared to sample values. Medians and quartiles deviations are greater from estimated distributions than from the sample, estimated moment characteristics of the level are close to mean and estimated standard errors are lower than sample standard deviation (not shown in the table). According to the logarithmic likelihood in the solution Dagum distribution is slightly superior to lognormal distribution (it is in accordance with literature dealing with incomes, Kleiber and Kotz, 2003).

Conclusion

In the text different equivalent units are used to evaluate equivalised incomes in the Czech Republic during six consecutive years 2006-2010. Equivalised incomes treated in the text are evaluated as a ratio of net annual total income of a household divided by number of equivalent units. From this definitions we can derive that these incomes are highly correlated for different definitions of equivalent units (in all years). High correlation coefficients were found above 0.8 for logarithms of incomes (because of positive skewness of all analysed incomes, Figures 1 and 2).

Above the known ordering in the analysed incomes (based on (6) and inequality $x \geq \sqrt{x}, x \geq 1)$ for all households, even sample and estimated characteristics are almost constant in all years when being given in per cents. From Table 3 it follows that for medians and quartile deviations (taking as 100 % values for equivalised income based on modified OECD-scale that is used in European Union) income per capita is 75 % (median), 70 % (q), for equivalised income with OECD-scale 88 % (median), 80 % (q) and for units based on squared root of number of members 107 % (median), 120 % (q). These values hold approximately for all years. For estimated medians and quartile deviations these values are for Dagum distribution 88 % (83 %), 75 % (75 %) and 107 % (115 %) and for lognormal similarly 88 % (86 %), 74 % (77 %) and 108 % (113 %).

From the point of view of probability theory these incomes are highly comparable with very similar properties. In (OECD, 2015) is stated that there is no generally useable definition and the suitable equivalence units should be selected based on economic decisions to meet its main purpose as well as possible.

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