SCENARIO ANALYSIS OF NON-LIFE TECHNICAL PROVISIONS: THE CZECH INSURANCE MARKET CASE STUDY

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Abstract

This contribution introduces and discusses a complex econometric model of non-life technical provisions based on the Czech non-life insurance market data. Selected economic-actuarial relations among given insurance variables are described by means of the dynamic linear system of simultaneous equations. In particular, the provision for outstanding claims, the provision for unearned premium, the other (marginal) technical provisions, the acquisition and administrative expenses, the benefit expenses, and their mutual interactions are studied in greater detail. The suggested econometric model is estimated, statistically verified, and interpreted with special regard to the actuarial point of view. The proposed modelling scheme can be further employed for prognosing the considered non-life technical provisions. Particularly, these forecasts can be taken into account by non-life insurance companies in their internal calculations (e.g. for financial planning purposes, for verifying the sufficiency of non-life technical provisions, or for liability adequacy tests LAT) or by an insurance regulator (e.g. for performing stress tests). Alternatively, this approach might motivate development of an internal model applicable in the Solvency II framework. Both deterministic and randomly generated scenarios are analysed; they are capable of delivering relevant outputs for formulating various recommendations and conclusions.

Key words: econometric system of simultaneous equations, non-life insurance, scenario analysis, Solvency II, technical provisions.

JEL Code: C30, C32, C39.

Introduction

Technical provisions are undoubtedly key insurance variables. They represent the amount of money maintained by an insurance company needed to meet all its future liabilities towards the clients (under a certain measurement of present obligations). The technical provisions must be sufficient to cover all these anticipated commitments at all times. It should be ensured by various regulatory principles introduced, e.g., by the Solvency I or Solvency II framework. The sufficiency of technical provisions is continuously monitored by national regulators and other supervisory authorities (e.g. in the Czech case by the Czech National Bank).

Generally, one distinguishes between the life and the non-life technical provisions (according to the underlying insurance contracts). All the provisions are regularly recalculated, verified, and reported in the annual (quarterly, monthly) balance sheets on the liability side. There exist several exactly specified categories of the technical provisions given by the legal framework of each country.

The present paper introduces and discusses a complex econometric model of the most significant non-life technical provisions based on the Czech non-life insurance market data. Particularly, the dynamic linear system of simultaneous equations is employed in order to describe different interactions among the selected economic-actuarial insurance variables. Namely, the provision for outstanding claims, the provision for unearned premium, and the other (marginal) technical provisions are studied in greater detail. After statistical verification, the suggested modelling scheme can be further applied for prognosing the considered non-life technical provisions. Particularly, these forecasts can be taken into account by non-life insurance companies in their internal calculations (e.g. for financial planning purposes, for verifying the sufficiency of non-life technical provisions, or for liability adequacy tests) and by insurance regulators (e.g. for performing stress tests or for verifying a prudency level). Alternatively, they might be useful for formulating an internal model applicable in the Solvency II framework.

Different aspects of econometric models, which investigated cash flows or technical provisions in the life insurance, were discussed in various academically or practically oriented works (Feilmeier & Junker, 1982; Cipra, 1998; Baranoff, 2007; Hendrych, 2011; Hendrych & Cipra, 2015). However, to the best of our knowledge there has not been published any complex econometric model examining non-life technical provisions (or even based on the Czech non-life insurance market data). On the contrary, several particular non-life technical provisions and related issues have been analysed in the literature from the statistical or actuarial points of view (Dahms, 2012; Hürlimann, 2009; and many others).

1 Model of non-life technical provisions for the Czech insurance market

As was mentioned above, we shall concentrate on the following three key categories of the non-life technical provisions: (*i*) the provision for outstanding claims, (*ii*) the provision for unearned premium, and (*iii*) the other (marginal) provisions (i.e. the sum of all other marginal non-life technical provisions representing only a minority of the total volume of all non-life provisions). The provision for outstanding claims is the estimated value of (future) compensations for policyholders and policy beneficiaries. More specifically, it involves the provision for IBNR (Incurred But Not Reported) claims and the provision for RBNS (Reported But Not Settled) claims. The provision for unearned premium corresponds to such a part of the written premium, which relates to future accounting periods. The other (marginal) provisions involve, e.g., the provision for bonuses and sales.

For simplicity, let us consider only relationships arising from the quarterly published summary balance sheets (the liability side) of all the Czech non-life insurance companies. Interactions among these accounting data might be further investigated through econometric modelling concepts based on the actuarial theory (Cipra, 2010). Nonetheless, one could possibly extend the introduced dataset by including other insurance or economic variables.

In particular, we assume the following non-life insurance variables: CS_t - the claims expenses in time t (in thousands of CZK), EAC_t - the acquisition expenses in time t (in thousands of CZK), EAD_t - the administrative expenses in time t (in thousands of CZK), EB_t - the existing business in time t (i.e. the number of existing non-life insurance contracts, in pieces), NRC_t - the number of reported claims in time t (in pieces), TPC_t - the technical provision for outstanding claims in time t (in thousands of CZK), TPO_t - the other non-life technical provisions in time t (in thousands of CZK), TPP_t - the technical provision for unearned premium in time t (in thousands of CZK), TPT_t - the total reported non-life technical provisions in time t (in thousands of CZK) defined as $TPT_t = TPC_t + TPO_t + TPP_t$, WP_t - the written premium in time t (in thousands of CZK), $t = 1, \dots, 28$ ($t = 1$ refers to the Q4 2008 and $T = 28$ to the Q3 2015). The quarterly based dataset was obtained from the quarterly reported summary balance sheets (the liability side) published by the Czech National Bank (ČNB) on the regular basis.[1]

We can proceed to the formulation of the dynamic linear econometric system of simultaneous equations, which describe relationships among the particular non-life insurance market variables listed above. The considered modelling scheme simultaneously explains the casual relations among more than one dependent variable. Therefore, it enables to model the analysed phenomenon in greater complexity. To be more precise, it reflects mutual

[1] www.cnb.cz/cnb/STAT.ARADY_PKG.STROM_DRILL?p_strid=BC&p_lang=CS, retrieved March 2, 2016.

interactions among the studied insurance variables through the following complex modelling structure (by assuming non-trivially correlated residuals).

We have considered the following simultaneous equations model (for t = 2, ..., T):

$$\begin{aligned}
TPP_t &= \beta_{11} + \beta_{21}1_{[Q2]} + \beta_{31}1_{[Q3]} + \beta_{41}1_{[Q4]} + \phi_{11}TPP_{t-1} + \beta_{61}\Delta WP_t^* + \varepsilon_t^{TPP}, \\
TPC_t &= \beta_{12} + \beta_{22}1_{[Q2]} + \beta_{32}1_{[Q3]} + \beta_{42}1_{[Q4]} + \phi_{22}TPC_{t-1} + \phi_{62}CS_{t-1}^* + \beta_{82}EB_t + \varepsilon_t^{TPC}, \\
TPO_t &= \beta_{13} + \phi_{33}TPO_{t-1} + \gamma_{53}\Delta EAD_t^* + \phi_{43}EAC_{t-1}^* + \beta_{73}NRC_t^* + \varepsilon_t^{TPO}, \\
EAC_t^* &= \beta_{14} + \phi_{14}TPP_{t-1} + \beta_{94}EB_{t-1} + \beta_{64}WP_{t-1}^* + \beta_{74}NRC_t^* + \varepsilon_t^{EAC}, \\
EAD_t^* &= \beta_{15} + \beta_{55}\Delta WP_t^* + \phi_{55}EAD_{t-1}^* + \beta_{85}EB_t + \phi_{35}TPO_{t-1} + \varepsilon_t^{EAD}, \\
CS_t^* &= \beta_{16} + \phi_{66}CS_{t-1}^* + \beta_{76}NRC_t^* + \beta_{56}WP_t^* + \beta_{86}EB_t + \varepsilon_t^{CS}, \\
TPT_t &= TPC_t + TPO_t + TPP_t,
\end{aligned} \tag{1}$$

where $1_{[\bullet]}$ denotes the binary indicator of the event •, Δ stands for the first difference operator, and the superscript * indicates that the variable has been seasonally adjusted (by using a simple routine multiplicative seasonal factor method). Moreover, βs, γs, and ϕs with various indices represent the parameters of the model and εs denote the stochastic error terms.

The considered dynamic econometric system (1) includes six stochastic equations (i.e. the equations with the stochastic residual terms) and one deterministic equation (i.e. the identity for the total provisions). In the suggested model, the intercept, the seasonal dummies, and the variables EB, NRC^*, WP^* (and thus also the lagged EB and WP^*) are assumed to be strictly exogenous (i.e. uncorrelated with residual components at all times); these variables enter into the system from outside. Such a particular choice of exogenous variables seems to be pragmatic with regard to the apparent external character of these variables. Furthermore, the lagged endogenous variables CS^*, EAC^*, EAD^*, TPC, TPO, and TPP are supposed to be predetermined (i.e. uncorrelated with current and future residual disturbances); they are fully determined by the system (1) in time t-1. Note that each equation in (1) satisfies the necessary condition of identification (Greene, 2003; Cipra, 2013).

To be precise, the model (1) (after ignoring the last deterministic equation) follows the structural form of the dynamic system of linear simultaneous equations (Greene, 2003):

$$\mathbf{y}_t^{\mathrm{T}}\mathbf{\Gamma} + \mathbf{y}_{t-1}^{\mathrm{T}}\mathbf{\Phi}_1 + \mathbf{x}_t^{\mathrm{T}}\mathbf{B} + \boldsymbol{\varepsilon}_t^{\mathrm{T}} = \mathbf{0}^{\mathrm{T}}, \tag{2}$$

where:

$$\begin{aligned}
\mathbf{y}_t &= (TPP_t, TPC_t, TPO_t, EAC_t^*, EAD_t^*, CS_t^*)^{\mathrm{T}}, \\
\mathbf{x}_t &= (1, 1_{[Q2]}, 1_{[Q3]}, 1_{[Q4]}, WP_t^*, WP_{t-1}^*, NRC_t^*, EB_t, EB_{t-1})^{\mathrm{T}}, \\
\boldsymbol{\varepsilon}_t &= (\varepsilon_t^{TPP}, \varepsilon_t^{TPC}, \varepsilon_t^{TPO}, \varepsilon_t^{EAC}, \varepsilon_t^{EAD}, \varepsilon_t^{CS})^{\mathrm{T}},
\end{aligned} \tag{3}$$

where $\mathbf{y}_t$ denotes the (6×1) vector of endogenous variables, $\mathbf{x}_t$ is the (9×1) vector of strictly exogenous variables, and $\boldsymbol{\varepsilon}_t$ stands for the (6×1) stochastic vector of the structural error terms (everything for all given times t). Point out that the indices of the parameters β, γ, and ϕ in (1) refer to the corresponding elements of the parameter matrices $\mathbf{B}$ (for strictly exogenous variables), $\mathbf{\Gamma}$ (for endogenous variables), and $\mathbf{\Phi}_1$ (for predetermined variables with the time lag 1), respectively. Moreover, some a priori constraints must be assumed, namely $\beta_{61} = -\beta_{51}$, $\gamma_{53} = -\phi_{53}$, $\beta_{65} = -\beta_{55}$. Other elements of the matrices of parameters are equal to zero. Several other assumptions are usually introduced: (*A1*) $\mathbb{E}(\boldsymbol{\varepsilon}_t) = \mathbf{0}$ for all t, $\mathrm{var}(\boldsymbol{\varepsilon}_t) = \mathbb{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t^{\mathrm{T}}) = \mathbf{\Sigma}$ is a symmetric positive definite matrix for all t, and $\mathrm{cov}(\boldsymbol{\varepsilon}_s, \boldsymbol{\varepsilon}_t) = \mathbb{E}(\boldsymbol{\varepsilon}_s\boldsymbol{\varepsilon}_t^{\mathrm{T}}) = \mathbf{0}$ for all $s \neq t$; (*A2*) $\mathbb{E}((\mathbf{y}_{t-1}^{\mathrm{T}}, \mathbf{x}_t^{\mathrm{T}})^{\mathrm{T}}(\mathbf{y}_{t-1}^{\mathrm{T}}, \mathbf{x}_t^{\mathrm{T}})) = \boldsymbol{Q}$ is a finite symmetric positive definite matrix for all t; (*A3*) the matrix $\mathbf{\Gamma}$ containing the parameters concerning (exclusively) endogenous variables is invertible with elements -1 on its diagonal (Greene, 2003; Lütkepohl, 2005).

To estimate the unknown parameters of the proposed system of econometric equations (1), the three-stage least squares method (3SLS) might be considered. This full information estimation technique is a special case of the generalized method of moments GMM exploiting all information available in the considered system. It guarantees suitable properties (under general assumptions). Namely, the 3SLS estimates are consistent, asymptotically normally distributed, and asymptotically efficient (Greene, 2003). Table 1 presents the 3SLS estimates of the model (1) jointly with the estimated standard errors.

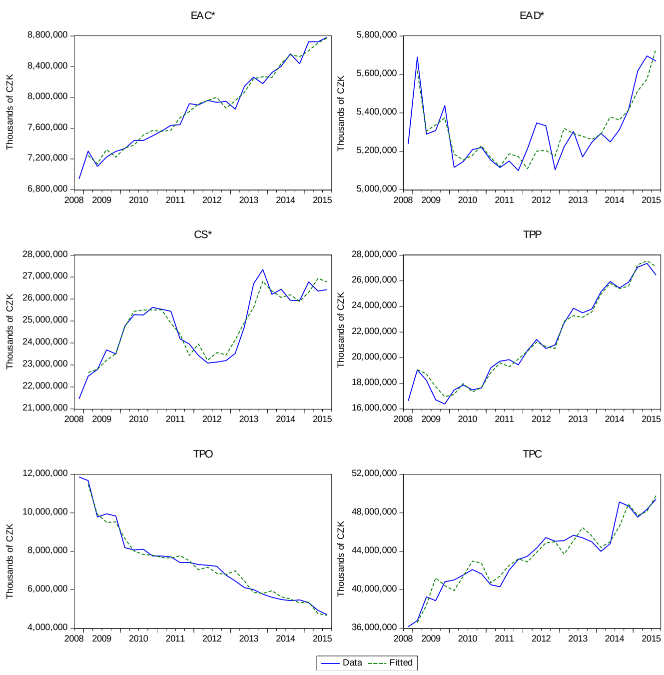
Tab. 1: The 3SLS estimates of the parameters of the model (1)

Eq. for TPP		***Eq. for TPC***		***Eq. for TPO***		***Eq. for EAC****		***Eq. for EAD****		***Eq. for CS****	
Par.	*Est.* *(Std. Err.)*	*Par.*	*Est.* *(Std. Err.)*	*Par.*	*Est.* *(Std. Err.)*	*Par.*	*Est.* *(Std. Err.)*	*Par.*	*Est.* *(Std. Err.)*	*Par.*	*Est.* *(Std. Err.)*
β_{11}	1467464 (451313)	β_{12}	2039251 (2796570)	β_{13}	14165828 (2355754)	β_{14}	4799534 (565896)	β_{15}	447354 (517488)	β_{16}	-6429938 (2272260)
β_{21}	-693205 (201651)	β_{22}	833827 (404924)	β_{73}	-4.48788 (1.47497)	β_{64}	-0.06710 (0.01208)	β_{55}	0.11712 (0.01581)	β_{56}	0.31614 (0.05910)
β_{31}	-1831019 (194997)	β_{32}	1437856 (405270)	γ_{53}	1.32742 (0.25785)	β_{74}	1.22320 (0.35485)	β_{85}	0.08230 (0.02454)	β_{76}	15.16822 (2.02663)
β_{41}	-1134179 (199541)	β_{42}	618533 (418687)	ϕ_{33}	0.54494 (0.06365)	β_{94}	0.12381 (0.02003)	ϕ_{35}	0.04138 (0.01479)	β_{86}	-0.55251 (0.10278)
β_{61}	0.37131 (0.08208)	β_{82}	0.92553 (0.25469)	ϕ_{43}	-0.89766 (0.19209)	ϕ_{14}	0.09357 (0.00774)	ϕ_{55}	0.56685 (0.10284)	ϕ_{66}	0.70127 (0.06974)
ϕ_{11}	0.98911 (0.02169)	ϕ_{22}	0.69778 (0.07629)								
		ϕ_{62}	-0.26473 (0.14110)								

Source: Authors (by EViews 8.0).

One can see that the estimated model fits the data suitably (see Figure 1). Moreover, it demonstrates its statistical adequacy (refer to the assumptions listed above). For instance, the sample correlation matrix of the estimated 3SLS residuals delivers several relatively high correlations (see the outputs displayed in Table 2). The Portmanteau tests and the empirical autocorrelation functions of the 3SLS residuals do not indicate the presence of residual autocorrelations (Lütkepohl, 2005). Furthermore, the joint Jarque-Bera test cannot reject the multivariate normality of the 3SLS residuals. Finally, neither the Hausman specification test comparing the two and the three-stage least squares estimates nor the Sargan test for overidentifying restrictions reject the proper model specification (Greene, 2003; Cipra, 2013). Note that the standard 5% significance level has been applied.

Fig. 1: The observed endogenous variables with their fitted counterparts



Source: Authors (by EViews 8.0).

Tab. 2: The estimated 3SLS residual correlation matrix

	TPP	*TPC*	*TPO*	*EAC**	*EAD**	*CS**
TPP	1.00000	0.26336	-0.39427	-0.01615	0.02415	-0.11868
TPC	0.26336	1.00000	-0.45937	0.06458	0.05888	-0.52206
TPO	-0.39427	-0.45937	1.00000	-0.09768	0.03929	0.32066
*EAC**	-0.01615	0.06458	-0.09768	1.00000	0.12879	0.17445
*EAD**	0.02415	0.05888	0.03929	0.12879	1.00000	-0.25189
*CS**	-0.11868	-0.52206	0.32066	0.17445	-0.25189	1.00000

Source: Authors (by EViews 8.0).

2 Scenario analysis

From the practical point of view, prognosing the discussed non-life technical provisions might be truly useful. For instance, one could employ randomly generated scenarios for stress testing the sufficiency of the particular non-life technical provisions by applying the modelling scheme (1). Consequently, this approach might motivate development of an internal model for non-life insurance companies, which might be introduced in the Solvency II framework in order to prescribe the solvency capital requirement SCR. Moreover, it could be used by any supervisory authority to set or revise the prudency level effectiveness.

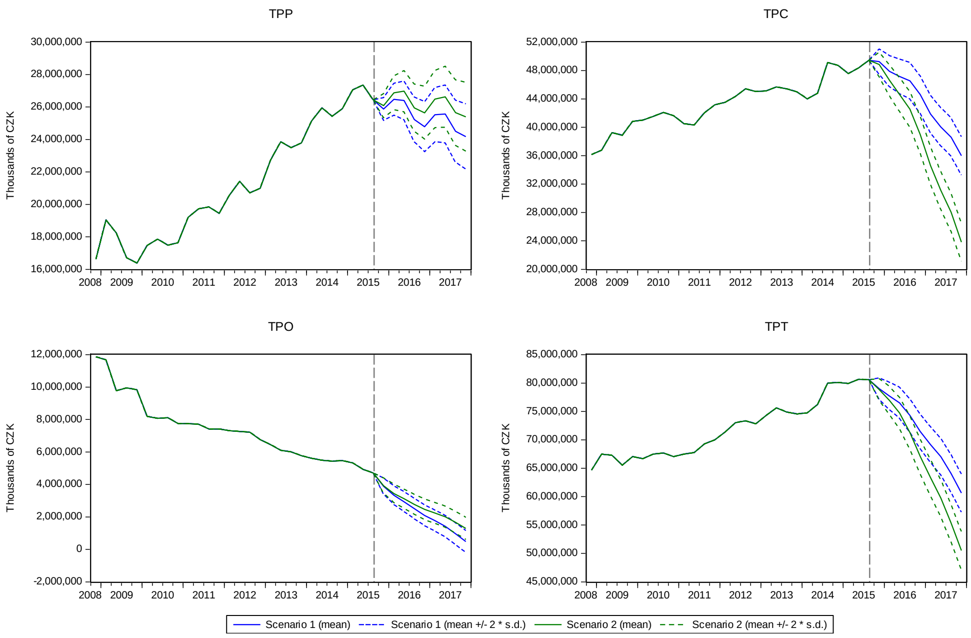
To illustrate the key idea of the stress testing discussed above, two different stress scenarios for the exogenous variables EB, NRC^*, and WP^* are considered. Both scenarios are rather pessimistic since one usually tests the sufficiency (or the prudency level effectiveness) of the provisions under (extremely) unfavourable conditions. The first scenario is formulated as follows: the number of existing non-life insurance contracts EB decreases by 3% each quarter, the number of reported claims (seasonally adjusted) NRC^* increases by 3% each quarter, and the written premium (seasonally adjusted) WP^* decreases by 5% each quarter, t = 29, …, 37, i.e. from Q4 2015 to Q4 2017. Note that the seasonally adjusted written premium decreases faster than the existing business. The second underlying scenario follows analogous expectations as before only with the minor changes: the number of existing non-life insurance contracts EB decreases by 5% each quarter, the number of reported claims (seasonally adjusted) NRC^* increases by 3% each quarter, and the written premium (seasonally adjusted) WP^* decreases by 3% each quarter, t = 29, …, 37, i.e. from Q4 2015 to Q4 2017. Here, the seasonally adjusted written premium decreases slower than the existing business portfolio.

Accepting these two underlying stress scenarios, we have further employed the suggested modelling scheme (1). Particularly, we have calculated 10000 realizations (forecasts) of all the endogenous variables for each given stress scenario and the whole prediction horizon by using the prescribed strictly exogenous variables EB, NRC^*, WP^* and 10000 randomly generated vector error terms $\boldsymbol{\varepsilon}_t$; refer to (1) and (2)-(3). We have applied the standard residual bootstrap method (Hendrych & Cipra, 2015). In particular, it means that the multivariate distribution of the disturbances $\boldsymbol{\varepsilon}_t$ has been determined by the (centred) empirical residuals computed during the realized 3SLS estimation. See Table 1. All computations were performed in EViews version 8.0 by authors' calculation procedures.

The results of simulation study are summarized in Figure 2 and Figure 3. At first sight, one can see that both scenarios have an impact on all the non-life technical provisions; all the presented provisions have significantly decreased. Nevertheless, it corresponds to rational anticipations for the considered scenarios. Furthermore, one can identify that the differences between both assumed stress scenarios are truly significant. Especially, let us compare the results of prognosing the provision for outstanding claims (*TPC*) in Figure 2. The mean forecasted provision is substantially lower for the second underlying stress scenario. However, it perfectly reflects the cumulative impact of the strictly exogenous variables on the particular non-life technical provisions (consult the model (1)). From the analysis of the cumulative impact, which goes beyond the scope of this paper, one might presume that the number of existing non-life insurance contracts (*EB*) and the number of reported claims (NRC^*) considerably influence the total non-life technical provisions (see Figures 2 and 3).

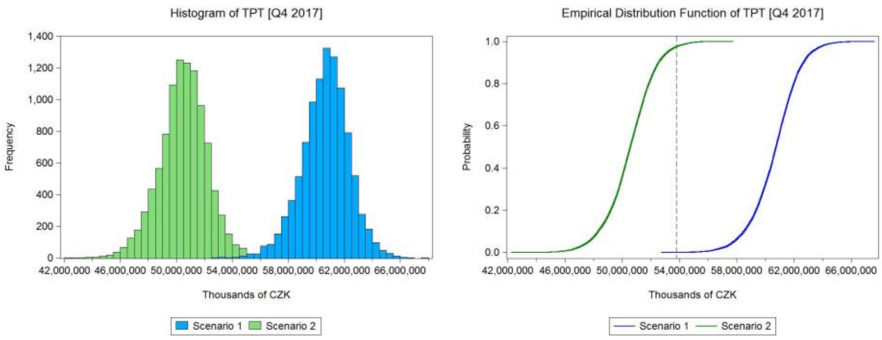
From the actuarial viewpoint, the generated projections provide several useful interpretations (see Figure 3). For instance, we can observe that the empirical probability that the total non-life technical provision *TPT* in Q4 2017 will be less or equal to the two thirds of the Q3 2015 level is only 0.05% for the first stress scenario but 97.46% for the second stress scenario, respectively. The substantial difference is apparent (refer to the comments above). Such outputs could be further employed, e.g., by the regulator for testing the sufficiency of the total provisions or for calibrating the prudency level.

Fig. 2: The results of prognosing the particular non-life technical provisions



Source: Authors (by EViews 8.0).

Fig. 3: The results of prognosing the total non-life technical provisions (Q4 2017)



Source: Authors (by EViews 8.0).

Conclusion

The contribution presented the complex econometric model of the key non-life technical provisions (and other important actuarial variables) based on the Czech non-life insurance market data. In particular, the econometric system of dynamic linear simultaneous equations was employed in order to describe economic-actuarial relationships within the quarterly published summary balance sheets of the Czech non-life insurers. The estimated modelling scheme was further employed in the stress testing analysis. It might be used, e.g., for financial planning purposes or for testing the sufficiency of the non-life technical provisions.

To illustrate the main idea of the discussed stress testing, two underlying (extremely) unfavourable scenarios of future development based on the prescribed strictly exogenous variables were investigated in detail. The simulation results evaluated by using the residual bootstrap corresponded to pragmatic anticipations (i.e. the decreasing tendency of all the particular non-life technical provisions). Consequently, one could identify that the number of existing non-life insurance contracts and the number of reported claims influenced the total non-life technical provisions substantially.

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