E-LEARNING AND FINANCIAL MATHEMATICS

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Abstract

Based on the National Strategy for Financial Education is now necessary to understand basic financial and mathematical relationships that relate and deal with the financial market and which is now slowly but surely becoming a necessity not only for employees of banks, insurance companies and brokerage companies, but at the moment almost everyone, we solve internal grant University of Economics and Management (VŠEM) Prague Financial Mathematics for Everybody with UEM. In this article we review the assumptions on which we based the project (tree sources – Mathematics for Everybody with UEM project, best textbooks on financial mathematics for university programs in economics, some achievements leads to incorrect application due to specificities of mathematics, including financial mathematics). From a series of project activities the following are the rich outputs of the project with an emphasis on elements of e-learning solutions of the project (tree parts), especially on-line application offering in terms of video presentations (or mini lessons), tests and Financial Mathematics for Everybody with UEM User Guide.

Key words: financial mathematics, Financial Mathematics for Everybody with UEM, e-learning, video presentations, tests

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Introduction

This article is a follow-up on Coufal, Tobíšek (2015) and Tobíšek, Coufal (2015).

The National Financial Education Strategy (Národní strategie finančního vzdělávání, 2010) defines financial literacy as follows: „*Financial literacy is a set of knowledge, skills and value statements of a citizen, necessary for him/her to support himself/herself and his/her family financially in contemporary society and to act actively on financial products and services market. A financially literate citizen has awareness of money and price issues and is able to take care of the family budget responsibly, including management of financial assets*

and financial liabilities with respect to changing life situations." We should note that it is one of the basic documents systematically defining financial education in the Czech Republic. Here, first the importance of financial education is described. Then follows a definition of financial literacy as the sum of necessary competencies. The document further lays out future priority areas and principles of financial education. The two-pillar structure of financial education is also clarified here. One pillar of financial education targets pupils in introductory education, the other pillar aims at educating citizens and is thus focused on adult population. Chapter five of this document is then devoted to the role of key subjects and coordination of activities in the field of financial education. The decisive document for schools is called System for Building Financial Literacy in Primary and Secondary Schools (Systém budování finanční gramotnosti na základních a středních školách, 2007), furthering National Financial Education Strategy and defining the target state of financial education for primary schools (separately for elementary and post-elementary level) and for secondary schools. Financial literacy is presented in terms of outcomes and contents of education in the following areas:

a) money,
b) household economy,
c) financial products,
d) consumer rights.

The contents and outcomes of education are then defined in these areas. Financial literacy is interdisciplinary in nature, positioned between social and economic sciences, mathematics and information technologies. It is about the basics of correct financial thinking and efficient economic behavior. Financial mathematics plays an important role in this respect.

The main classical disciplines of mathematics developed from four practical human needs – the need to count when making trade, the need to understand relations between amounts provided in numbers, the need to measure land plots and constructions, and the need to predict astronomical phenomena. From these four needs four classical mathematical disciplines evolved – arithmetic, algebra, geometry, and mathematical analysis, dealing more or less with four basic areas of interest of mathematics – quantity, structure, space, and change. Mathematics had started with numbers and numbers are still of fundamental importance for it, although for a long time now it has not been limited to numerical calculations. The pioneers of mathematics have achieved significant breakthroughs, but have also faced impasses (sometimes even for centuries!), but that is the pioneering way. Good mathematical ideas only seldom grow out of fashion, although their implementation can change dramatically.

Mathematics usually works in hiding or "backstage" and it would be very easy to fall for a misguided feeling that there is no real work of mathematics. That is unfortunate, as it makes us think technologies work by a miracle and it makes us expect new miracles every day.

To understand the basic financial and mathematical relations relating to and dealing with financial market has slowly but steadily become a necessity not only for employees of banks, insurance companies, and investment companies, but also for virtually all of us. The correct answers can also be found via financial mathematics, i.e. mathematics applied in the area of finance (financial market, its subjects and tools), although applications of mathematics are being used not only in science, industry, or trade, but even in art. The trade routes from China to Europe or from Indonesia to America have held together by an imaginary mathematical thread. Current society could never function without mathematics. Mathematics is universal and omnipresent.

A completely lay person imagines columns of numbers, numerous logarithmic, interest rate or insurance tables, and sets of various statistical data as financial mathematics. More experienced observer, on the other hand, tends to liken financial mathematics and the multitude of its formulae to a coffee mill: you put some data in, turn the crank a bit, and there comes the desired result. Financial mathematics is none of that. The formulae are a means, not an end. They are an embodiment of thought economy, and they serve us to avoid the necessity of re-thinking everything "from Adam on" again and again. Any science or scholarship (including finance) forces us to define new terms and create new theories. The aim is to tear down the wall of ambiguity so often standing in the way of scholarly progress. Here the role of mathematics and logic is irreplaceable. As "financial mathematics" we label mathematical operations in financial sphere. Older literature also uses the term "political arithmetic." Financial mathematics is part of larger applied mathematics and deals with mathematical tasks related to financial calculations and mathematical models of financial markets. In financial mathematics we study a financial instrument from the perspective of a (possibly random) cash flow generated by this instrument.

If we go *ad fontes* (to the roots) we can divide financial mathematics into four parts:

a) classical financial mathematics – percentage calculations, questions concerning debt instruments (bill of exchange, depository certificate, bond) and also analysis of financial flows in the world of banking (credit loan, investment),

b) stochastic financial mathematics – studies financial markets and tries to find a fair (or non-arbitrary) price of financial instruments using stochastic calculus,
c) insurance mathematics – actuary calculations, i.e. calculations used in insurance industry,
d) econometric calculations – prognoses of future development of financial markets.

Engineers and various ingenious persons create tools and instruments, but it is up to the academics to write treatises on those; hence, it will be needed to slightly modify the historic interpretations of growth, innovations and numerous similar phenomena. In financial mathematics students are expected to work systematically, and yet systematic work is not on par with current lifestyle and possibilities offered to young people.

1 Three sources

The aim of the project is to provide an interesting form of high-quality education in financial mathematics, thus to increase financial literacy and to promote financial mathematics as scholarly discipline among those interested.

In order to motivate it is however necessary to get properly accustomed to the sense and procedure of implementation and to benefits of new technological solutions. The teaching does not serve technologies here: the technologies are to serve teaching. The outcome of the project will provide students with sophisticated and fun learning method, leading them to see financial mathematics as a tool to understand the world of finance and not as a set of formulae and theorems. When conceiving the grant project and implementing its first part we built on the National Financial Education Strategy (Národní strategie finančního vzdělávání, 2010) and to enhance the System for Building Financial Literacy in primary and secondary schools (Systém budování finanční gramotnosti na základních a středních školách, 2007) we have created a draft approach, methods, content and technology for Financial Mathematics course at UEM.

We have supposed that the course will target all interested internet users and hence we have designed the on-line applications as both tools for direct teaching of financial mathematics and as self-standing educational tools that can also serve to prepare for an exam in financial mathematics, especially for part-time students. We also suppose it will be used for vocational training and re-training courses. The methods, content and technology, as well as this article, have been prepared based on three sources.

1.1 Source one

To implement the project in both technological and methodical terms we drew on experience from Mathematics for Everybody with UEM project (Matematika VŠEM, 2016), more specifically from its sub-project Promotion of Mathematics and Support for Transition of High-school Students to Technology-focused Universities (reg. no. CZ.2.17/3.1.00/36239), implemented as part of the Operational Programme Prague Adaptability.

Current students use modern technologies in much of their free time and they are accustomed to use this technology not only to communicate among themselves, but also with other people. They follow the latest trends in technology and can easily find their way around them and use them. Easy access to technology and its understanding as a regular part of life allows high school students to use these tools not only for free-time activities, but also for other activities of their lives (such as study or short-term jobs).

Both mathematics and financial mathematics are peculiar subjects of study. Hence the video presentations can neither have the form of capturing the exposition in classroom, nor that of a movie showing the lecturer with only small part of the screen containing the exposition. It is thus necessary to simulate the development of solution as if on a whiteboard and hence to use a classic animated PowerPoint presentation accompanied by spoken text.

1.2 Source two

First we dealt with assumptions on the level of knowledge in financial mathematics among high school students Clark, Brechner (2007) and Kaspříková, Klůfa (2014). This sources are used for lesson repetition. Another good resources are the possibly most suitable and (currently) best Czech textbooks on financial mathematics for university programs in economics by Cipra (2005 and 2010) and Radová, Dvořák, Málek (2013) that we used to design not just the course curriculum, but also substantial part of its content. There is a slight issue with ambiguous Czech terminology in financial mathematics, hence we prefer the terms from Cipra (2005).

1.3 Source three

To avoid errors in using e-learning we used experience formulated in Mošna (2006) and Klůfa (2013 and 2016) when preparing outcomes of the project. These articles show that using some

achievements leads to incorrect application due to specificities of mathematics, including financial mathematics.

2 Three parts

Based on the target group we created a syllabus using Cipra (2005 and 2010) and Radová, Dvořák, Málek (2013):

1. Basic terms (percentage count, functions, averages, sequences and series, money, time, interest rate, interest, equity, loans, debt, securities, investment).
2. Simple interest (current account and authorized overdraft, interest number and interest divisor, short-term securities, discounting).
3. Compound interest (compound interest, discount, mixed interest, short-term compound interest, continuous interest, real interest rate, annual percentage rate of charge, i.e. APR).
4. Amortization of debt (amortization of debt by unequal payments, amortization of debt by equal payments, amortization plan).
5. Regular and fixed payments (regular payments in interest period, current and final value of fixed payment, savings, various tasks with fixed payments, some specific types of fixed payments).
6. Obligations (basic terms and types of obligations, price and earning capacity of obligations until maturity, earnings curve, duration).
7. Time-value of money and investment decision-making (income tax, depreciation, value equation, current value, inner earning capacity, recoverability time, criteria of investment decision-making, portfolio analysis).

This syllabus is an Ariadne's thread for creating outcomes of the Financial Mathematics for Everybody with UEM project, mainly including:

a) video sequences (or video presentations) where we explain the terms, relations among them, and solve topical problems (we use available software to solve more difficult tasks, showing the process in MS Excel applying functions in this software), and where we show animation of the algorithm in question,
b) knowledge tests – hypertext links showing 5 random examples of problems from database relating to the topic,
c) Financial Mathematics for Everybody with UEM User Guide.

These outcomes are located at Mathematics for Everybody with UEM website (Matematika VŠEM, 2016).

2.1 Part one – video presentations

We used not only experience with e-learning at UEM, but also the possible pros and cons of some elements of e-learning Mošna (2006) and Klůfa (2013 and 2016). We simulate the development of solution as if on a whiteboard and thus we decided to use classic animated PWP presentation with text presented visually being accompanied by spoken commentary. Individual video presentations are minilessons and we aim to keep their timeframe between 10 and 25 minutes.

The software used to create PWP presentations and entire video presentations including sound is identical to the one used in Mathematics for Everybody with UEM (Matematika VŠEM, 2016).

To solve more difficult calculations we use not only financial, but also mathematical (SUM, POWER etc.) and statistical (MEAN, GEOMEAN etc.) functions in MS Excel, as we consider this software to be the most available one for target audience, i.e. students of non-economical and economical (but not financial) college programs as well as trainees in professional and requalification training courses.

In terms of graphics each page of any project presentation includes the Financial Mathematics for Everybody with UEM project logo.

Video presentations can be found on the Internet at Mathematics for Everybody with UEM website (Matematika VŠEM, 2016) under the Financial Mathematics menu.

2.2 Part two – tests

We opted for multiple-choice tests, but also the possible pros and cons of some elements of multiple-choice tests Klůfa (2013 and 2016), Kaspříková, Klůfa (2014). The "Knowledge Tests" menu at Mathematics for Everybody with UEM website (Matematika VŠEM, 2016) includes hypertext links showing at random 5 examples of problems from database relating to the topic. With each problem 5 or more possible outcomes are provided, again in random order. Thus if the user generates multiple sets of problems from the same topic, the correct answer does not necessarily have the same label. The user first enters his/her e-mail address

for unique identification of the test-taker, then he/she marks those solutions of individual problems that he/she considers correct based on his/her calculations. After having marked the last solution the user clicks on the "Send the test and show results" option. The system evaluates correctness of the answers and shows results. The first part of the results shows the overall assessment in percentage as arithmetic mean of success from all test questions scores. For example, if the user solved one problem at 100 % and all the other four at 0 %, the resulting overall assessment is 20 %. Assessment of individual question reflects the correctness of answers to all the options provided (not marking any option results in failed answer). If there is more than one correct answer and the user only picks one, the resulting assessment of the question is amended in proportion. As for the examples of problems used, their difficulty is chosen to allow for solution in MS Excel. Examples of problems are as follows:

a) A person offers his bank a bill of exchange to discount, the bill being mature in ¾ of a year with nominal value of 180 000 CZK. The bank discounts the bill at discount rate of 1.9 % p.q. How much will the person get in cash for the bill?
b) If I regularly, anticipatively and annually deposit 132 000 CZK on a deposit book with interest of 1.25 % p.a. and compound interest running, without any other manipulation with the deposit, how much will I have on my deposit book in 8 years (i.e. at the beginning of the 9th year)? Do not forget to consider a 15 % interest tax.

2.3 Part three – Financial Mathematics for Everybody with UEM User Guide

The Tools (Pomůcky) menu at Mathematics for Everybody with UEM website (Matematika VŠEM, 2015) will also include Financial Mathematics for Everybody with UEM User Guide in pdf format and an overview of financial mathematics formulae. Currently this menu includes two other guides: Mathematics for Everybody with UEM – Methods for High-school Teachers and Mathematics for Everybody with UEM – Project User Manual (in Czech). We use experience with both of these Manual when creating the User Guide, setting its contents as follows: (a) Introduction; (b) Videopresentations with Succint Contents; (c) Overview of Formulae Used in Teaching and Necessary for the Tests; (d) Index.

Conclusion

Some say that financial mathematics is like a mouse trap. We can put it otherwise. Financial mathematics is like an ocean, and the one who once dares to sail it either gets sea sick,

thinking with dread of its depth and width, or gets infatuated in its endless waters. That is why financial mathematics is such a great adventure of thought.

We believe that the study of project outcomes will lead to furthering both knowledge and skills of users in financial mathematics. They will for instance understand the mechanism of using call option on futures, for this financial operation had been used already in Antiquity, as a well-known story shows:

It is said that one of the seven sages of Ancient Greece, the traveler and worldly merchant Thales of Miletus (about 640-562 BC) was often ridiculed for being poor and for uselessness of philosophy. Watching the stars he discovered that there will be a huge yield of olives. Already in winter he acquired some money, paid deposits for all olive presses in Miletus and on Chios and rented them for low price, as nobody else offered more. When the harvest time came, everyone needed the presses and he was able to rent them at any price he wanted, so he accumulated huge amount of money. In today's terms he used a call option on futures. He thus also showed that it is easy for philosophers to get rich, but it is not what they struggle for.

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