EVALUATION OF PROBABILITY OF MARKET STRATEGY PROFITABILITY USING PCA METHOD

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Abstract
Principal component analysis (PCA) is a commonly used and very powerful method for simplification of multivariate data. In the presented contribution, it is, however, utilized in a novel way. The methodology described in the paper provides tables of probabilities of market strategy profitability. Values of the first two principal components are utilized in evaluation of conditional probability. The resulting probability tables can help to detect buying signals for profitable trade opportunities or can provide a warning against a high financial loss.

A time period of two weeks before an actual buying signal is analyzed. After one week of holding, the share is sold and the resultant relative price differences determine the group of profitability (high profit, profit, loss, high loss). For each group, tables of occurrences and conditional probabilities are evaluated using the PCA method. The whole methodology is tested on a case study that takes in account one year long dataset (1st Aug 2014 – 14th Aug 2015) of L-3 Communications Holding shares.

Key words: profitability, conditional probability, principal component analysis

JEL Code: C38, G11, G17

Introduction
Analysis of the financial market is very large topic and one needs a lot of knowledge and experience for its full understanding you. Most of the investors, however, do not attempt to fully understand all the analyses and they prefer to use one or two methods that are considered most effective (Lo et al., 2000), (Neely et al., 1997), (Byun et al., 2015). Each method is based on a different approach and has various prerequisites for successful investing. Nevertheless, all methods have one thing in common; they are trying to understand the current situation in the market and to predict whether the prices or shares on financial market will grow or decline. According to Bauer and Dahlquist (1998), the main motto of the majority of trades is "cheap
acquisition of a financial asset or his precious sales". Based on scientific studies Lee et al. (2017), Lynch et al. (2000), Lo et al. (2000) and Neely et al. (1997), all kinds of statistical methods are applied nowadays to build reliable prediction model, e.g. principal component analysis (PCA). So we can conclude that prediction of prices future development is a constantly evolving area.

PCA is a popular method in the area of financial analyses; in particular it is used to define and to quantify the factors that influence changes in the prices of securities or shares (Gemperline, 2006). Utilization of PCA for evaluation of profit probability is, however, novelty in this research field. The prerequisite is classification of profit levels for selected trading strategy so each trading day, consequently, belong to one profit class. Based on data precedent selected trading day, principal components are evaluated. As a result from whole dataset, we are getting conditional probabilities of profit class occurrences based on principal component values. Proposed methodology is similar to method for prediction of wind ramp events, which was presented and validated in (Heckenbergerova, 2014), (Heckenbergerova, 2015) and (Heckenbergerova, 2016).

This paper consist five main sections. After introduction, principal component analysis is briefly described. Second section summarizes methodology that is used in case study presented in the section 3. Major conclusions with direction of future work are provided in the last section.

1 Principal Component Analysis

The basic idea of principal component analysis (PCA) is the reduction of the number of variables by using the so-called the principal components. Each principal component is formed as linear combination of the original variables and it is characterized by covariance of all the variables. Therefore most significant principal components can summarize knowledge about the default variables while preserving all the original information (Lo et al., 2000), (Neely et al., 1997), (Gradojevic and Lento, 2015).

According to Rencher (2002), PCA is based on a covariance matrix of the \( p \) variables \( X_j \). From its eigenvalues and eigenvectors, new variables \( Z_j \) (principal components) are derived that are uncorrelated and sorted according to decreasing variance (eigenvalues).
Let us assume that all data from selected dataset are forming $N$-dimensional random vector $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_N)$ and moreover let us consider the time frame of length $n$. Observation matrix $\mathbf{X}$ is then in form:

$$\mathbf{X} = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_n \\ Y_2 & Y_3 & \cdots & Y_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N-n} & Y_{N-n+1} & \cdots & Y_N \end{bmatrix},$$

(1)

where each row corresponds to one time frame of simulation and each column is a random vector $\mathbf{X}_j$ of given time index.

In the next step eigenvalues and eigenvectors of covariance matrix $\text{var} \, \mathbf{X}$ are evaluated using singular value decomposition:

$$\text{var} \, \mathbf{X} = \mathbf{S} \mathbf{V} \mathbf{D}'$$

(2)

where $\mathbf{S}$ is an orthogonal $(n \times n)$-matrix with columns of singular vectors, $\mathbf{V}$ is a diagonal $(n \times n)$-matrix corresponding to eigenvalues of $\mathbf{X}$ and columns of matrix $\mathbf{X} \mathbf{D}'$ are the principal components of matrix $\mathbf{X}$ (Wall et.al, 2003).

2 Methodology

Methodology of evaluation of conditional probability tables can be summarized in following steps:

1) Dataset (time series of share prices) selection;
2) Market strategy selection (holding period);
3) Sorting of trading days based on profitability (creation of profitability classes);
4) Creation of observation matrix $\mathbf{X}$ with selected length of time frame prior trade;
5) Evaluation of principal components;
6) Division of principal components ranges to create support rectangles and table axes;
7) Evaluation of occurrence frequencies on each support rectangle from whole dataset and from each profitability class;
8) Evaluation of conditional probability for each support rectangle and creation of conditional probability tables.

Resulting tables show us conditional probability of the future trade profitability based on actual principal component values.
3 Case study

1.1 Data and Market Strategy

Let us take in account the one year long dataset of L-3 Communications Holding shares (1\textsuperscript{st} Aug 2014 – 14\textsuperscript{th} Aug 2015). Our market strategy is selling the share after one week of holding (5 business days). For each trading day relative profitability ($RP$) is evaluated using formula:

$$RP = \frac{(P_{sell} - P_{buy})}{P_{buy}},$$

(3)

where $P_{buy}$ is buying price for selected trading day and $P_{sell}$ is share price on selling day. Based on these values, data are divided into four groups: HP (high profit) for profitability greater than 0.5; P (profit) for $RP$ values between 0 and 0.5; L (loss) for negative profitability between -0.5 and 0; and HL (high loss) for profitability below -0.5 threshold. Table 1 summarizes the size of profitability classes and Figure 1 shows time series of L-3 Communications Holding shares with high profit and high loss trades highlighted. All figures are generated in Matlab using our own scripts.

Tab. 1: Profitability classes

<table>
<thead>
<tr>
<th>Type of class</th>
<th>Size of class</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole dataset</td>
<td>262</td>
</tr>
<tr>
<td>HP (high profit)</td>
<td>43</td>
</tr>
<tr>
<td>P (profit)</td>
<td>84</td>
</tr>
<tr>
<td>L (loss)</td>
<td>106</td>
</tr>
<tr>
<td>HL (high loss)</td>
<td>23</td>
</tr>
</tbody>
</table>

Source: own
Fig. 1: Share prices of L-3 Communications Holding from 1st Aug 2014 till 14th Aug 2015 augmented with high profitable (triangle up) and high loss (triangle down) trades

Source: own

1.2 Principal Component Analysis

Let us assume that all data from selected dataset are forming \( N \)-dimensional random vector \( Y = (Y_1, Y_2, \ldots, Y_N) \) and moreover let us consider the time frame of length \( n = 10 \), corresponding to two weeks horizon before buying signal (10 business days). Observation matrix \( X \), based on (1), has then 10 columns and 252 rows.

In the next step eigenvalues and eigenvectors of covariance matrix \( \text{var } X \) are evaluated using singular value decomposition. As a result we are getting principal components of observation matrix \( X \). First three principal components (PCA1, PCA2 and PCA3) are illustrated in Figure 2, where color and shape of triangles signalize different profitability classes (green-HP, yellow-P, magenta-L, red-HL). Components PCA1 and PCA2 can explain together 39\% respectively 52\% of the total variability of data, therefore they are chosen as significant ones.
Fig. 2: Three principal components for L-3 Communications Holding dataset

Source: own

1.3 2D Tables of Trade Profit Conditional Probabilities
Let us divide the PCA axes into 12 parts so that the whole graph area is covered by 2D blocks (rectangles). Frequency tables for HP, P, L and HL events are created by the number of occurrences in the corresponding 2D block. They are captured in following figure 3.

Fig. 3: Frequencies of profit classes occurrence on selected 2D blocks.
Conditional probability value in selected rectangle for any profitability class is defined as fraction of profitability class frequency and all data frequency in that block. Figure 4 shows column graphs of evaluated conditional probabilities.

We can see that for high value of first principal components PCA1, there is significant probability (close to one) that buying now and selling in a week will generate high loss. We can also conclude, that low values of PCA1 (less than 30) shows very low risk of high loss.

**Fig. 3: Conditional probabilities of trade profitability given by the 2D block of principal components**
Conclusion

This paper presents a principal component analysis of share prices time series to evaluate probability of selected market strategy profit. Illustrated methodology uses share prices of L-3 Communications Holding from 1.August 2014 till 14.August 2015. Market strategy is selling the share after one week of holding. Two weeks horizon before actual buying signal has been analyzed to see whether trade profitability is predictable.

Principal components, frequencies of occurrence and conditional probabilities of trade profitability have been evaluated using described methodology. Gained results show that low values of first principal component are not in favor with trade with high loss. In contrary, high values of PCA1 will result in high loss with high probability. Therefore the resulting probability tables can help to detect future buying signals for profitable trade opportunities or can provide a warning against a high financial loss.

Our goal in our further research is validation of presented methodology. More and current datasets will be taken in account and validation will be performed on last quarter of selected databases.

References


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