HOW TO APPROXIMATE THE IDEAL FISHER PRICE INDEX?

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Abstract

The Consumer Price Index (CPI) approximates changes in the costs of household consumption assuming the constant utility. In practice, the Laspeyres price index is used to measure the CPI, despite the fact that many economists consider superlative indices to be the best approximation of Cost of Living Index (COLI). The Fisher index is the most popular among superlative indices and it is called "ideal" since it satisfies most of tests derived from the axiomatic price index theory, including time reversibility. Nevertheless, the Fisher price index makes use of current-period expenditure data, and thus its usefulness in CPI measurement is limited. In this paper, we verify the utility of using the Lloyd-Moulton and AG Mean indices for the Fisher price index approximation. We confirm this utility in an empirical study based on the latest data for the European Union countries and for different levels of data aggregation (3-digit and 4-digit COICOP level is considered).

Key words: Consumer Price Index, Fisher Index, AG mean index, Lloyd-Moulton Index

JEL Code: C10, C43, E31,

Introduction

The consumer price index (CPI) measures changes in the price level of market basket of consumer goods and services purchased by households. The CPI is a statistical estimate constructed using the prices of a sample of representative items whose prices are collected periodically and it approximates changes in the costs of household consumption assuming the constant utility (COLI, *Cost of Living Index*). In practice, the Laspeyres price index is used to measure the CPI (White,1999; Clements & Izan, 1987). The Lapeyres formula does not take into account changes in the structure of consumption which occur as a result of price changes at a given time interval. It leads to the conclusion that the Laspeyres index can be biased due to the commodity substitution. Many economists treat *superlative indices* (such as the Fisher index or the Törnqvist index) as the best approximation of COLI (Von der Lippe, 2007). The difference between the Laspeyres index and the superlative index should approximate the value of the commodity substitution bias. The Fisher index is the most popular among superlative indices and it is called "ideal" since it satisfies most of tests derived from the axiomatic price index theory, including time reversibility. Nevertheless, the Fisher price index makes use of current-period expenditure data, and thus its usefulness in CPI measurement is limited. In this paper, we verify the utility of using the Lloyd-Moutlon and AG Mean indices for the Fisher price index approximation. We confirm this utility in an empirical study based on the latest data for the European Union countries and for different levels of data aggregation.

1 The Fisher price index

Let us consider that we observe N commodities from the CPI basket of goods during the time interval $[s,t]$, where t denotes the current period and s denotes the base period. Let us denote by $P^{\tau}=[p_1^{\tau}, p_2^{\tau},..., p_N^{\tau}]^T$ the vector of N considered prices at any moment τ and by $Q^{\tau}=[q_1^{\tau}, q_2^{\tau},..., q_N^{\tau}]^T$ the vector of N considered quantities at any moment τ. The CPI is a Laspeyres-type index defined by

$$P_{La}=\frac{\sum_{i=1}^{N} q_i^s p_i^t}{\sum_{i=1}^{N} q_i^s p_i^s}, \qquad (1)$$

so we assume here the constant consumption vector at the base period level. The Fisher price index (P_F) is a geometric mean of the Laspeyres (P_{La}) and the Paasche indices (P_{Pa}), i.e.

$$P_F=\sqrt{P_{La}P_{Pa}}, \qquad (2)$$

where the Paasche index formula is as follows

$$P_{Pa}=\frac{\sum_{i=1}^{N} q_i^s p_i^t}{\sum_{i=1}^{N} q_i^s p_i^s}, \qquad (3)$$

2 The Lloyd-Moulton price index

The quadratic mean of order r price index was defined in (Diewert, 1976) as follows ($r \neq 0$)

$$P_{QM}(r) = \{[\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{\frac{r}{2}}]^{\frac{2}{r}} [\sum_{i=1}^{N} w_i^t (\frac{p_i^t}{p_i^s})^{-\frac{r}{2}}]^{-\frac{2}{r}}\}^{\frac{1}{2}}, \tag{4}$$

where w_i^s and w_i^t denote the expenditure share of commodity i in the base period s and the current period t, respectively. It is a superlative price index. By setting $r = 2(1-\sigma)$ expression (4) becomes

$$P_{QM}(\sigma) = \sqrt{P_{LM}(\sigma) \cdot P_{CW}(\sigma)}, \tag{5}$$

where $P_{LM}(\sigma)$ denotes the Lloyd-Moutlon price index defined as (Lloyd, 1975; Moulton, 1996; Shapiro & Wilcox 1997)

$$P_{LM}(\sigma) = [\sum_{i=1}^{N} w_i^s (\frac{p_i^t}{p_i^s})^{1-\sigma}]^{\frac{1}{1-\sigma}}, \tag{6}$$

and $P_{CW}(\sigma)$ denotes its “current weight (CW) counterpart”, namely

$$P_{CW}(\sigma) = [\sum_{i=1}^{N} w_i^t (\frac{p_i^t}{p_i^s})^{-(1-\sigma)}]^{\frac{-1}{1-\sigma}}. \tag{7}$$

For $\sigma = 0$, we obtain $P_{LM}(0) = P_{La}$, $P_{CW}(0) = P_{Pa}$ and $P_{QM}(0) = P_F$. Since the price index $P_{LM}(\sigma)$ monotonically decreases and $P_{CW}(\sigma)$ monotonically increases as σ increases (see Biggeri, Ferrari (2010)), we conclude that there exists a value σ_0 which satisfies

$$P_{LM}(\sigma_0) = P_{CW}(\sigma_0) = P_{QM}(2(1-\sigma_0)). \tag{8}$$

Thus, we get that for $\sigma = \sigma_0$ the Lloyd–Moulton index becomes superlative and the found parameter is then the approximated elasticity of substitution. From Diewert (1976) we know that superlative indices approximate each other. The immediate conclusion is that using the Constant Elasticity of Substitution (CES) framework, a superlative Fisher price index can be approximated once we have estimated the elasticity of substitution. The Lloyd–Moulton price index does not make use of current-period expenditure data, so it is even possible to approximate a superlative index in real time and extrapolate the time series (Feenstra & Reinsdorf, 2007; Biggeri & Ferrari, 2010;Greenlees, 2011; Armknecht & Silver, 2012; Białek, 2015). The value σ_0 should be obtained numerically.

3 The arithmetic-geometric average index

In (Lent & Dorfman, 2009) it is proved that a weighted average of the Laspeyres index and the geometric Laspeyres index can approximate the Lloyd-Moulton index, and thus it also approximates the superlative target index. In particular, the above-mentioned weighted average (called the AG Mean index) provides a close approximation to the Fisher price index, namely

$$P_F \approx P_{AG} = \sigma \prod_{i=1}^{N} \left(\frac{p_i^t}{p_i^s}\right)^{w_i^s} + (1-\sigma) \sum_{i=1}^{N} w_i^s \left(\frac{p_i^t}{p_i^s}\right). \tag{9}$$

Solving the equation $P_F = P_{AG}$ with respect to σ , we obtain (Armknecht and Silver, 2012)

$$\hat{\sigma} = \frac{P_F - P_{La}}{P_{GLa} - P_{La}}, \tag{10}$$

where P_{GLa} denotes the geometric Laspeyres price index (Von der Lippe, 2007)

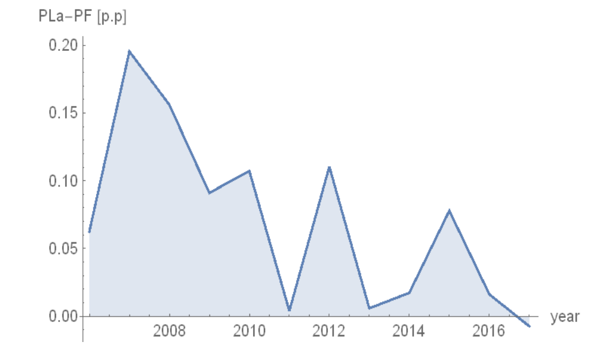
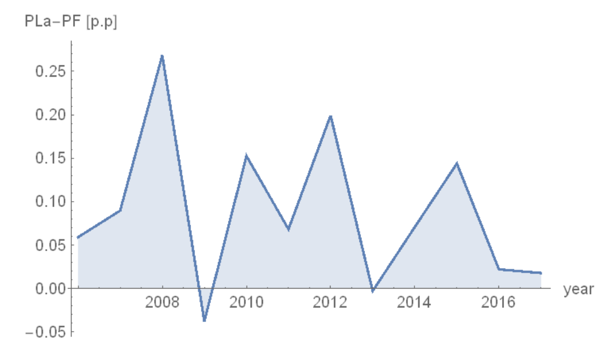
$$P_{GLa} = \prod_{i=1}^{N} \left(\frac{p_i^t}{p_i^s}\right)^{w_i^s}. \tag{11}$$

Since the parameter should not change rapidly over time, we can estimate it using historical data. Apparently, we should have a good tool for the approximation of the current value of the Fisher price index. In the empirical study, we verify the utility of the AG Mean index in the Fisher price index approximation using HICP data (3-digit and 4-digit COICOP level) from the EU countries.

4 The empirical study

As mentioned in Section 3, yearly HICP data (3-digit and 4-digit COICOP level) were used to assess the suitability of the AG index and the Lloyd-Moutlon index in the Fisher index approximation. The data concerned the EU countries and the period 2005-2016, which enabled the calculation of the Laspeyres index since 2006. The commodity substitution bias is probably more visible in domestic CPI measurements, however, in the case of the data analysed, the maximum difference between the Laspeyres and Fisher indices amounted to as much as 0.96 percentage point for the studied period. The difference for the Czech Republic is presented in Fig. 1.

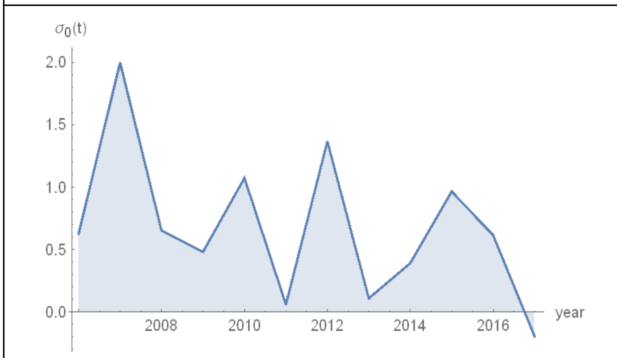
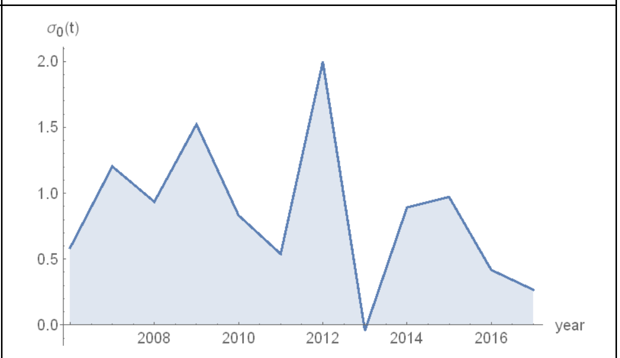
Fig. 1: Differences between the Laspeyres and the Fisher price indices for the Czech Republic in the years 2006-2017.

3-digit COICOP LEVEL	4-digit COICOP LEVEL
	

Source: own calculation in Mathematica 11.

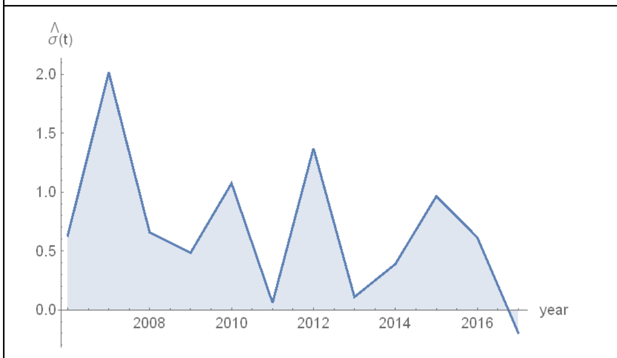
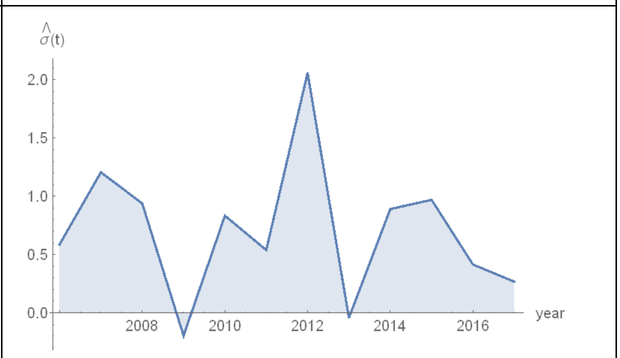
Estimates for σ_0 and $\hat{\sigma}$ for data from the Czech Republic and for each year from the considered interval are presented in Fig. 2 and Fig. 3. Histograms for these estimates calculated for the whole European Union and for 2017 are presented in Fig. 4 and Fig. 5.

Fig. 2: Estimates for σ_0 for data from the Czech Republic

3-digit COICOP LEVEL	4-digit COICOP LEVEL
	

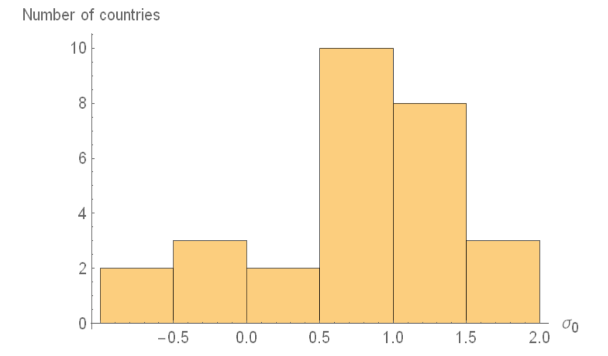
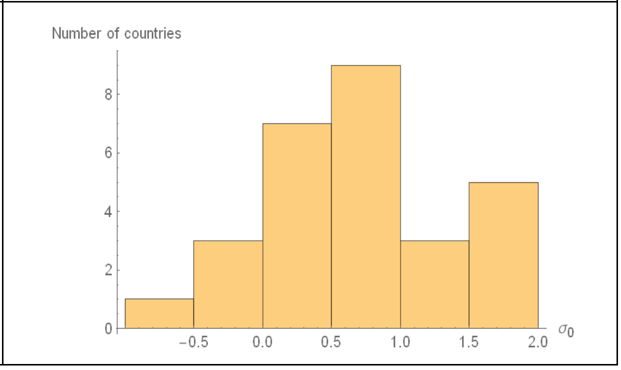
Source: own calculation in Mathematica 11.

Fig. 3: Estimates for $\hat{\sigma}$ for data from the Czech Republic

3-digit COICOP LEVEL	4-digit COICOP LEVEL
	

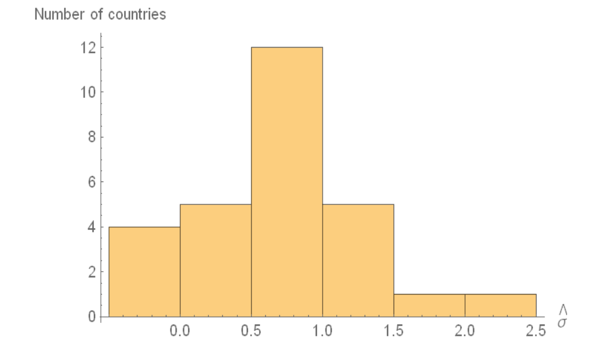
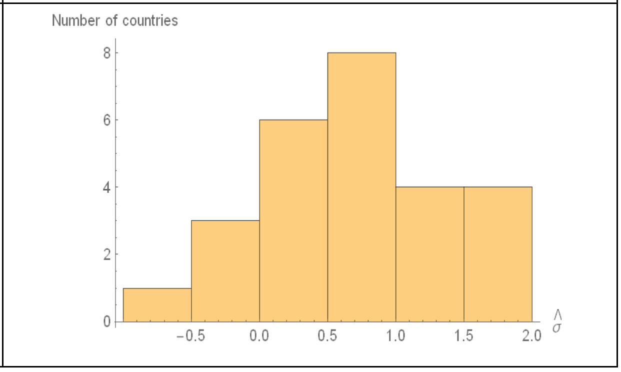
Source: own calculation in Mathematica 11.

Fig. 4: Histogram for σ_0 estimates among the EU countries in 2017

3-digit COICOP LEVEL	4-digit COICOP LEVEL
	

Source: own calculation in Mathematica 11.

Fig. 5: Histogram for $\hat{\sigma}$ estimates among the EU countries in 2017

3-digit COICOP LEVEL	4-digit COICOP LEVEL
	

Source: own calculation in Mathematica 11.

Conclusions

The example of the Czech Republic and many other EU countries indicates that the commodity substitution bias in the case of HICP is small, but that was the idea behind the creation of the HICP in the first place – the reduction of the said bias. However, this bias (calculated as the difference between the values of the Laspeyres and Fisher indices) is still not eliminated in the case of HICP (cf. Fig. 1), and the assessment of its value to a small extent, but, nevertheless, depends on the level of aggregation of data used for the calculation. The almost complete reduction of the substitution bias when measuring inflation, i.e. the ideal approximation of the Fisher index by means of an index with baseline weights, is possible but requires the estimation of the relevant parameters: σ_0 or $\hat{\sigma}$ (see Section 2 and 3).

Based on the example of the Czech Republic data, we can see that these parameters, although it seems that they should not change rapidly over time, are characterised by large fluctuations (see Fig. 2 and Fig. 3). This is a major drawback of the approximation methods discussed, as it is not possible to recommend a reference, fixed value of these parameters which would always ensure the best approximation of the Fisher index. On the basis of the same figures, it would also seem that the values σ_0 and $\hat{\sigma}$ determined based on the same data are similar (after all, they have the same economic interpretation!). However, comparing histograms in terms of the values of these parameters determined for all the EU countries (Fig. 4 and Fig. 5), we can clearly see differences, especially at the 3-digit COICOP level. The common denominator of all the four presented histograms, however, is the fact that using the frequentist criterion they recommend a $[0.5,1]$ range of these parameters.

In conclusion, it seems that for the application of the AG Mean index and the Lloyd-Moutlon index to be effective in practice, it is necessary to be able to predict the current value of parameters σ_0 and $\hat{\sigma}$ on the basis of at least several years of historical observation. The issue of utility of the discussed methods is therefore still open.

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