

# ANALYSIS OF THE RESULTS OF MATH TESTS AT THE UNIVERSITY OF ECONOMICS IN PRAGUE

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## Abstract

Analysis of the results of entrance examinations in mathematics at the Faculty of Business Administration at University of Economics in Prague is in present paper. Especially, the differences of number of points in the test in mathematics between test variants are studied in the paper. The differences may arise due to the varying difficulty of test variants, but also because of the different level of knowledge of students who write these variants. The problem of the homogeneity of the test variants (the varying difficulty of test variants) we shall describe in the following text. To increase the homogeneity of the test variants, the project Internal development competition number IRS/MF/F4/3/2016 was solved in 2016. The paper contains an evaluation of the results of this project. For the analysis of differences of number of points in the test in mathematics between test variants we shall use different statistical methods (analysis of variance, Bartlett's test etc.).

**Key words:** Entrance exams in mathematics, differences between test variants, analysis of variance.

**JEL Code:** C12, I21

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## Introduction

Entrance exams tests at University of Economics in Prague are the multiple choice question tests. The multiple choice question tests are suitable for admission process at the university. These tests are objective, results can be evaluated easily for large number of students. Disadvantage of the tests - student can obtain certain number of points in the test purely by guessing the right answers (for probability aspects of the multiple choice question tests see e.g. (Zhao, 2006), (Klůfa, 2015b)).

Entrance examinations tests in mathematics at the Faculty of Business Administration at University of Economics have 10 questions for 5 points and 5 questions for 10 points (100 points total). Questions are independent. Each question has 5 answers (one

answer is correct), wrong answer is not penalized – see e.g. (Klůfa, 2015a). Therefore, the number of points in the test in mathematics can be 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, . . . ,90, 95, 100.

The aim of this paper is to study the differences of number of points in the test in mathematics between test variants at University of Economics. This problem was solved e.g. in (Klůfa, 2016a), (Otavová and Sýkorová, 2016), (Klůfa, 2016b). To increase the homogeneity of the test variants, the database of math examples created by the Department of Mathematics was extended and divided into multiple groups in 2016. This problem was solved in the framework of the project Internal development competition number IRS/MF/F4/3/2016 - see (Klůfa, 2016c). Due to evaluation of the results of this project we shall study the differences of number of points in the test in mathematics between test variants at the Faculty of Business Administration before the project (in 2015) and after the project (in 2017). Similar problems are described in (Zvára and Anděl, 2001), (Otavová and Sýkorová, 2014), (Hrubý, 2016), (Löster and Langhamrová, 2012), (Klůfa, 2015c), (Bartoška, Brožová, Šubrt and Rydval, 2013), (Ječmínek, Kukulová, Moravec and Filipová, 2018). The results obtained in the paper will be used to further improve of the preparation of test variants in mathematics at University of Economics in coming years.

## 1 Differences between test variants before the project

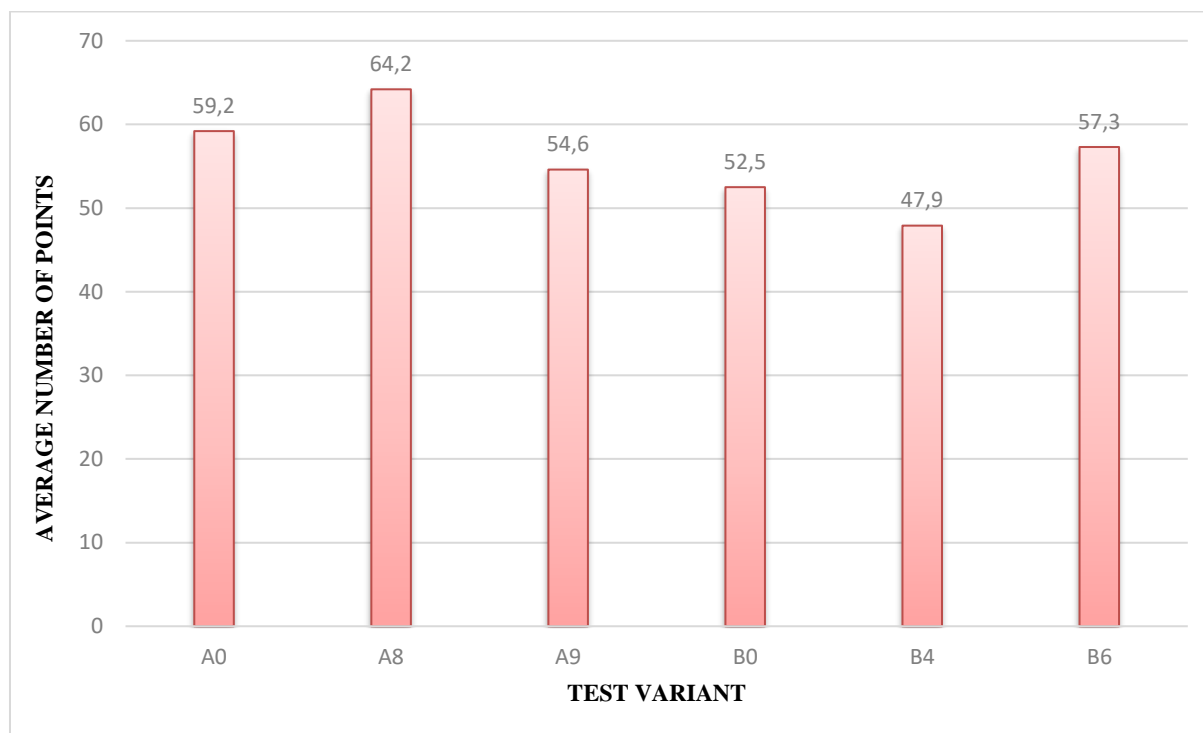
Six test variants were used for entrance exams in mathematics in 2015 at the Faculty of Business Administration at University of Economics in Prague. The basic descriptive characteristic of the distribution of number of points in the test in mathematics for these test variants are in Tab. 1.

**Tab. 1: Basic descriptive statistics for number of points in the math test in 2015**

Test variant	Frequency	Sum	Average number of points	Variance
A0	317	18775	59.23	543.942
A8	114	7315	64.17	540.229
A9	318	17365	54.61	559.703
B0	327	17180	52.54	584.028
B4	113	5415	47.92	544.967
B6	325	18625	57.31	462.714

Source: Own calculation

**Fig. 1: Average number of points in the math test in 2015**



Source: Own construction

The differences between average number of points in mathematics in test variants A0, A8, A9, B0, B4 and B6 (see Tab.1 and Fig. 1) could arise randomly (the maximum difference between averages is  $64.17 - 47.92 = 16.25$ ). For objective decision we shall use appropriate statistical test. We shall test null hypothesis

$H_0$ : the differences between average number of points in mathematics  
in test variants in 2015 are not statistical significant.

The decision reject or not reject this null hypothesis we can made using ANOVA or using the nonparametric Kruskal-Wallis test. Since differences between variances (see the last column of Tab. 1) are not statistical significant (it confirms Bartlett's test – see e.g. (Anděl, 1978)), basic assumption of ANOVA is fulfilled, therefore we shall decide about the validity of the null hypothesis  $H_0$  using ANOVA. The results of ANOVA are in Tab. 2.

**Tab. 2: Results of ANOVA**

Source of variability	Sum of Squares	Degrees of freedom	Fraction	F	P value	F crit
Test variants	23365.02	5	4673.004	8.682	3.99E-08	2.220
Residual	811706.1	1508	538.2667			
Sum	835071.2	1513				

Source: Own calculation

Since value of statistic  $F=8.682$  (for statistic  $F$  see e.g. (Anděl, 1978)) exceeds the critical value  $F_{0.05}(5,1508) = 2.220$  of Fischer-Snedecor distribution with 5 and 1508 degrees of freedom at significance level 0.05, i.e.

$$F=8.682 > F_{0.05}(5,1508) = 2.220,$$

the hypothesis  $H_0$  is rejected at significance level 0.05. Moreover,  $P$  value is  $4 \times 10^{-8}$  ( $H_0$  is rejected at significance level 0.01 and much less), so it is almost certain that between average number of points in mathematics in test variants in 2015 (see Fig. 1) are statistical significant differences.

## 2 Differences between test variants after the project

The results of 1297 students in entrance exams in mathematics in 2017 at the Faculty of Business Administration are analysed in this section. Six test variants were used for entrance exams in mathematics in 2017. The basic descriptive characteristic of the distribution of number of points in the test in mathematics for these test variants are in Tab. 3.

**Tab. 3: Basic descriptive statistics for number of points in the math test in 2017**

Test variant	Frequency	Sum	Average number of points	Variance
A2	156	8375	53.69	523.262
A3	138	7250	52.54	450.090
A7	355	17580	49.52	561.352
B0	156	7920	50.77	543.921
B1	352	17225	48.93	476.996
B2	140	7015	50.11	560.240

Source: Own calculation

The maximum difference between averages in 2017 is  $53.69 - 48.93 = 4.76$  (the maximum difference between averages in 2015 is 16.25). We shall test once again null hypothesis (see Fig. 2)

$H_0$ : the differences between average number of points in mathematics in test variants in 2017 are not statistical significant.

To verify the validity of the hypothesis we shall use once again ANOVA. In the first step we shall verify assumption of ANOVA (the same variance of number of points in test variants A2, A3, A7, B0, B1, B2) by Bartlett's test - see e.g. (Anděl, 1978). The hypothesis "variance of number of points in test variants A2, A3, A7, B0, B1, B2 is the same" is not rejected at 5% significance level, the assumption of ANOVA can be considered to have been met. The results of ANOVA we can find in Tab. 4.

**Tab. 4: Results of ANOVA**

Source of variability	Sum of Squares	Degrees of freedom	Fraction	<i>F</i>	P value	<i>F crit</i>
Test variants	3377.407	5	675.4814	1.299	0.262	2.221
Residual	671093.1	1291	519.8242			
Sum	674470.5	1296				

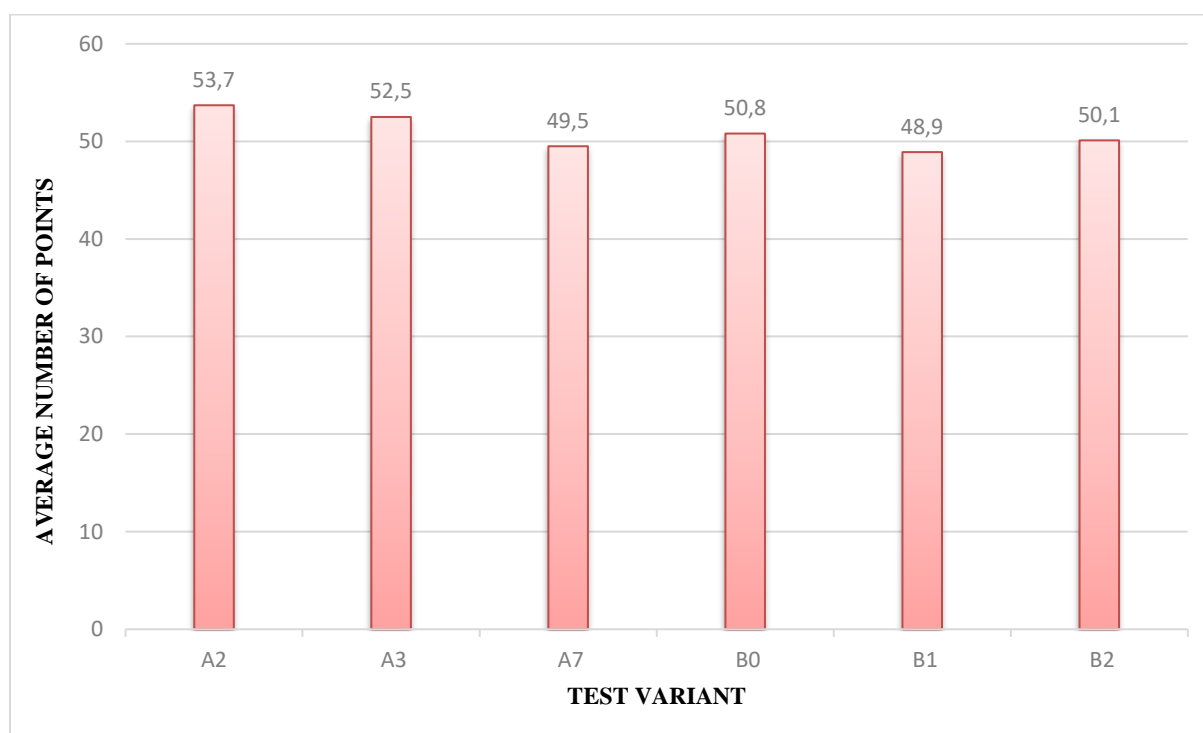
Source: Own calculation

Value of the statistic *F* is  $F=1.299$  (for statistic *F* see e.g. (Anděl, 1978)), the critical value of Fischer-Snedecor distribution with 5 and 1291 degrees of freedom at significance level 0.05 is  $F_{0.05}(5,1291) = 2.221$ . Because

$$F=1.299 < F_{0.05}(5,1291) = 2.221$$

the hypothesis  $H_0$  is not rejected at significance level 0.05. Moreover, *P* value is 0.262, it means that  $H_0$  is not rejected also at significance level e.g. 0.25. Between average number of points in mathematics in test variants in 2017 (see Fig. 2) are not statistical significant differences.

**Fig. 2: Average number of points in the math test in 2017**



Source: Own construction

## Conclusion

To increase the homogeneity of the test variants in entrance examinations in mathematics, the project Internal development competition number IRS/MF/F4/3/2016 at University of Economics in Prague was solved in 2016. The results of this paper show that the project has been fully met. The differences between average number of points in mathematics in test variants in 2015 (before the project) are statistical significant while the differences between average number of points in mathematics in test variants in 2017 (after the project) are not statistical significant. Moreover variance between test variants in 2017 is 675.4814 (see Tab. 4 – Fraction) and variance between test variants in 2015 is 4673.004 (see Tab. 2 – Fraction), i.e. variance between test variants in 2017 is much smaller than variance between test variants in 2015. The project contributed to increasing the objectivity of the admission procedure at the University of Economics in Prague.

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