A NOVEL NONLINEAR NEURAL NETWORK BASED ON COEFFICIENT OF VARIATION FOR SOLVING CARDINALITY CONSTRAINT PORTFOLIO OPTIMIZATION PROBLEM

Ilgim Yaman – Turkan Erbay Dalkilic

Abstract

Today, determining stocks in the portfolio is the major problems in the finance world. In 1952, Harry Markowitz had proposed standard portfolio optimization which is cornerstone of portfolio optimization. Mainly, in the Portfolio optimization problem main goal is minimizing the risk, while maximizing the expected return of portfolio. Since portfolio optimization problem is an NP-hard problem, hard computing techniques does not meet today's conditions. Due to time constraints and the necessity of economic situations, many heuristic methods were used to solve portfolio optimization method such as particle swarm optimization, ant colony optimization etc. In this study, Markowitz's mean-variance portfolio optimization with cardinality constraint is tried to solve which is not only quadratic optimization problem but also it is a binary integer programming problem. In order to solve mixed-integer quadratic optimization problem, we suggested nonlinear neural network based on coefficient of variation for solving cardinality constraint portfolio optimization (CCPO) problem. While analyzing the proposed algorithm efficiency, ISE-30 data (İstanbul Stock Exchange 30) was used between 10.06.2015-14.05.2017. Finally, the obtained results from the proposed algorithm are compared with the results obtained from the classic portfolio selection models in the literature.

Key words: Nonlinear neural network, cardinality constraint, portfolio optimization

JEL Code: C 45, C61, G11

Introduction

Markowitz (1952) had proposed portfolio theory which is cornerstone of modern portfolio theory. In the portfolio theory the most important issue that risk-return trade-offs. Besides, investors want to take minimum risk, they want to maximize expected return. In addition to these objectives, realistic management decisions reveal the necessity of cardinality constraints which is number of holding assets in the portfolio. In other words, investors want to limited the number of stocks that's why Mean-Variance cardinality constraint portfolio optimization (MVCCPO) problem is proposed (Chang, Meade, Beasley, Sharaiha, 2000). Since then, this problem has become quite popular in the world of finance and optimization. MVCCPO problem is mixed-integer optimization problem which makes it NP-hard (non-deterministic polynomial time hardness) problem. NP-hard exact algorithms have tried to solve MVCCPO problem such as branch-and-cut algorithm, branch and bound algorithm, difference of convex algorithm etc. Nevertheless, exact algorithms have disadvantages about high computation time and finding just exact solution, heuristic techniques can get ideal solution in sensible time.

In the literature, heuristic techniques have used to solve MVCCPO problems such as Differential evolution, evolutionary algorithm, tabu search algorithm, genetic algorithm, simulated annealing etc. In this study, we proposed nonlinear neural network to solve mean variance cardinality constraint portfolio optimization problem.

1.1 Mean-variance cardinality constraint portfolio optimization problem

The main difference in mean-variance cardinality constraint portfolio optimization is addition of cardinality constraint which denotes by z_i. That constraint limits the number of stocks in the portfolio. If the stock is not in the portfolio z_i have to be 0. That means fitness function also must be zero. In the beginning, investor can decide the number of stocks and expected return which are denoted by $K$and R^*.

The cardinality constraint problem has an integer type constraint, so portfolio optimization problem became mixed-integer quadratic optimization problem. Mean-variance cardinality constraint portfolio optimization problem can be formalized by:

$$min \sum_{i=1}^{N}\sum_{j=1}^{N} z_i x_i \sigma_{ij} x_j z_j$$

$$\text{s.t. } \sum_{i=1}^{N} x_i = 1$$

$$\sum_{i=1}^{N} z_i = K \qquad (1)$$

$$\sum_{i=1}^{N} x_i \mu_i = R^*$$

$$i = 1, \dots N$$

$$z_i \epsilon \{0,1\}$$

N: the number of assets available
z_i: binary variable about holding assets or not
x_i: investment propotion of i^{th} asset
σ_{ij}: covariance between asset i and j
K: desired number of assets in the portfolio
μ_i: expected return of i^{th} asset
R^*: expected return of portfolio

If the asset is held $z_i = 1$ otherwise $z_i = 0$. Expected return of portfolio, risk of portfolio and Sharpe Ratio denotes with respectively;

$$E(R_p) = \sum_{i=1}^{N} x_i \mu_i \qquad \sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}} \qquad S_p = \frac{E(R_p) - R_f}{\sigma_p}, \tag{2}$$

2. Nonlinear neural network for Portfolio Optimization Problem

Nonlinear neural network proposed by Nyguen (2000) to solve linear programming models. This non lineer neural network solves the primal and dual problem simultaneously. The nonlinear neural network consists of two layers. One of that is primal neuron and the other one is dual neuron. Inputs of primal neurons are outputs of dual neurons and their derivatives.

Yan (2014) extended Nguyen’s neural network to solve quadratic programming problems. The progress of in nonlinear neural network is given in such articles (Tank,1986; Ma, 1992; Xia,1996; Malek 2005). In 2019, portfolio selection based on nonlinear neural network is proposed by Yaman & Dalkilic (2019). Nevertheless, in Markowitz mean-variance model is no limitation about number of stocks. Cardinality constraint portfolio optimization is limited the number of stocks by cardinality constraint. In this study, we decide to limited cardinality constraint by using Inverse of coefficient of variation.

3. Application of Nonlinear Neural Network Approach to Cardinality Constraint Portfolio Optimization Determined by Inverse of Coefficient of Variation

Portfolio optimization with cardinality constraint makes the optimization problem mixed integer constraint portfolio optimization problem. It has become NP-hard problem which have been solved with heuristic techniques up to now. In this study, we propose to use inverse coefficient of variation in order to select stocks which have minimum risk and maximum expected return. In MVCCPO problem investors have to select K stocks which is the cardinality constraint frequently taken as a K=5 in the literature. Proposed method starts with calculating inverse of coefficient of variation for each stock. In the second step, half of stocks eliminate which have minimum inverse of coefficient of variation. Thus, half of stocks are hold. All 5 combinations of selected stocks are taken into nonlinear neural network. The expected return, risks and Sharpe ratio of portfolio were calculated.

In order to analyze the proposed algorithm efficiency, ISE-30 data (İstanbul Stock Exchange-30) was used between 10.06.2015-14.05.2017. In order to control results following 3-months data was used between 14.05.2017-16.08.2017. Finally, the obtained results from the proposed algorithm are compared with the results obtained from the standard portfolio optimization, simple nonlinear neural network for Portfolio optimization in the literature. In the first step inverse of coefficient of variation is calculated. Stocks which are bigger than the median of inverse of cardinality constraint are hold. Thus, 15 stocks are eliminated. In the second step all 5 combinations are taken into nonlinear neural network. In Table 1, results of proposed algorithm and comparative results of other two algorithms are given. In this study our main purpose is limiting stocks, in other words, main goal is solving cardinality constraint portfolio optimization.

Tab.1: Distribution of Stocks by Nonlinear Neural Network Approach to CCPO Problem as Determined by Inverse Coefficient

Stocks	Active Set Method	Nonlinear neural network	Nonlinear neural network with C^{-1}
ARCLK	0.0135	0	0.11
ASELS	0.0600	0	0.52
BIMAS	0.1869	0.35	0
DOHOL	0.0368	0.07	0
ECILC	0	0.02	0.17
ENKAI	0.1514	0.01	0
OTKAR	0.1065	0.08	0.14
PETKM	0.1267	0.34	0
TKFEN	0.0210	0.13	0
TUPRS	0.1019	0	0.05

TTKOM	0.0780	0	0
TCELL	0.1173	0	0
Expected Return	0.00094	0.00200	0.00200
Risk	0.00100	0.01231	0.01358
Sharpe Ratio	0.04163	0.16246	0.14661
f(x)	0.00005	0.00015	0.00018

Source: ISE-30 data (İstanbul Stock Exchange-30) was used between 10.06.2015-14.05.2017.

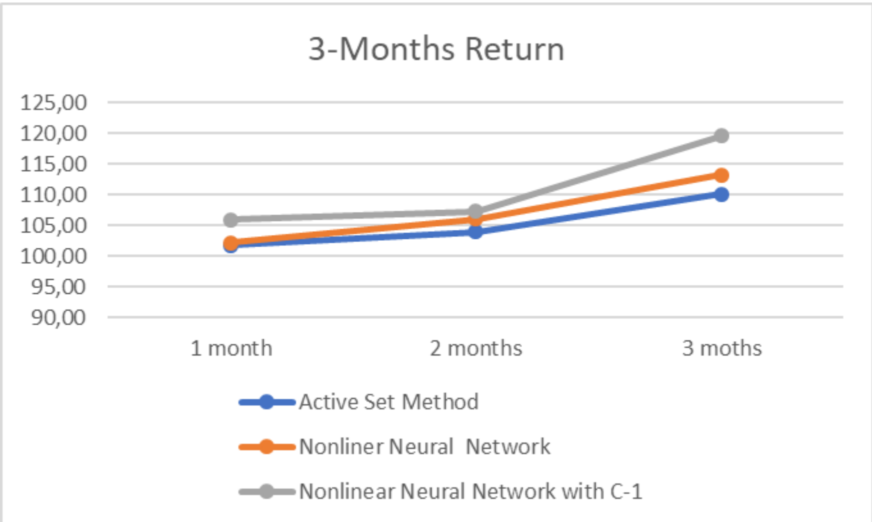
In the proposed method, selected stocks are as follows: ASELS %52, ECILC%17, OTKAR 14%, ARCLK %11 and TUPRS %5. In order to compare expected returns and the real life results the following 3 months data is taking in the consideration. In proposed portfolio, while one-month investment $100 becomes $105.9, two-month investment resulted in $ 100 to $ 107.3. Finally, three-month investment resulted in 119.6. The return of nonlinear neural network with C-1 is greater than the other methods as given Table 2. Moreover, as shown in Figure 3 the proposed method gets highest return in each month.

Tab. 2: 3-Month results when 100 $ is invested

Months / Methods	1st month	2nd months	3rd moths
Active Set Method	101.7830	103.9325	110.1131
Nonlinear Neural Network	102.1708	106.0168	113.1753
Nonlinear Neural Network with C-1	105.9046	107.3328	119.5700

Source: ISE-30 (İstanbul Stock Exchange-30) data was used between between 14.05.2017-16.08.2017

Fig. 1: 3-month comparison of 3 methods



Source: ISE-30 (İstanbul Stock Exchange-30) data was used between between 14.05.2017-16.08.2017

Conclusion

In this paper, nonlinear neural network with inverse of coefficient of variation is used to solve the mean-variance cardinality constraint portfolio optimization problem. Actually, nonlinear neural network solves mean variance portfolio optimization problem by selecting 7 stocks without limitation in number of stocks. Where, mean variance cardinality constraint model allows limiting the number of stocks in the portfolio, inverse of coefficient of variations provides profitable investments in proposed method. Result of 1 to 3-month investment is gets profitable results in proposed method.

References

1. Markowitz, H. (1952). Portfolio selection. *The journal of finance*, 7(1), 77-91.
2. Chang, T. J., Meade, N., Beasley, J. E., & Sharaiha, Y. M. (2000). Heuristics for cardinality constrained portfolio optimisation. *Computers & Operations Research*, 27(13), 1271-1302.
3. Tank, D., & Hopfield, J. J. (1986). Simple'neural'optimization networks: An A/D converter, signal decision circuit, and a linear programming circuit. *IEEE transactions on circuits and systems*, 33(5), 533-541.
4. Maa, C. Y., & Schanblatt, M. A. (1992). A two-phase optimization neural network. *IEEE Transactions on Neural Networks*, 3(6), 1003-1009.
5. Xia, Y. (1996). A new neural network for solving linear programming problems and its application. *IEEE Transactions on Neural Networks*, 7(2), 525-529.
6. Malek, A., & Yari, A. (2005). Primal–dual solution for the linear programming problems using neural networks. *Applied Mathematics and Computation*, 167(1), 198-211.
7. Nyguen, K.V. (2000).A Nonlinear Network for Solving Linear Programming Problem s. In*: International Symposium on Matematical Programming, ISMP*, Atlanta, GA, USA.
8. Yan, Y. (2014, November). A new nonlinear neural network for solving QP problems. *In International Symposium on Neural Networks* (pp. 347-357). Springer, Cham.
9. Yaman, I., & Dalkılıç, T. E. (2019). Portfolio selection based on a nonlinear neural network: An application on the Istanbul Stock Exchange (ISE30). *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics*, 68(2), 1709-1723.

Contact

Ilgım Yaman
Giresun University
Giresun University, Faculty of Arts and Science, Department of Statistics, Gure Campus, 28200, Giresun, Turkey
ilgim.yaman@giresun.edu.tr

Türkan ERBAY DALKILIC
Karadeniz Technical University
Karadeniz Technical University, Faculty of Science, Statistics and Computer Science, 61080 Trabzon/TÜRKİYE
terbay@ktu.edu.tr