## DECOMPOSITION OF MIXTURES OF PROBABILITY DISTRIBUTIONS IN THE PROBLEM OF EVALUATING THE INTEGRATION ACTIVITY OF VARIOUS SECTORS OF THE RUSSIAN ECONOMY

## Mariia Karelina – Tatiana Ivanova – Violetta Trofimova

#### Abstract

Statistics of mergers and acquisitions (M&A statistics) is a traditional direction for Western and international statistics and a new direction for the Russian statistical science. In accordance with the recommendations of Eurostat and the Organization for Economic Cooperation and Development, in order to improve business statistics in Russia, comprehensive information is needed on the integration activity of economic entities in various sectors of the Russian economy.

The statistical analysis of integration processes and major trends in the development of the Russian market of mergers and acquisitions in the context of the introduction of economic sanctions and import substitution has been conducted; a modern econometric tool for assessing the integration activity of the Russian economy based on parametric modelling is proposed. The analysis results of the development main directions of the Russian market of mergers and acquisitions in the conditions of global transformation processes have been generalized.

The proposed econometric tooling for classifying the sectors of the Russian economy by the level of integration activity based on the use of the model for the splitting of probability distributions mixtures made it possible to quantify sectors disproportions and identify the class of integrative-active sectors of the national economy.

**Key words:** decomposition of mixtures of probability distributions, integration activity, mergers and acquisitions, parametric modeling

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### Introduction

In recent decades, the role of economic integration of any state in the system of economic relations has significantly increased. The scope and the level of economic integration are substantially macroeconomic indicators of the effective functioning of the national economy

and its institutions (Barkoulas, 2001; Boateng, 2014). Integration processes contribute to restoring the structural integrity of the national economy, equalizing the spatial characteristics of the country's industrial potential, increasing the competitiveness of do-mestic products in present-day Russian conditions, which is the key to the re-industrialization of the domestic economy (Bersenev, 2003). The main causes of mergers and acquisitions are the economies of scale achieved with new technologies, the entry of financial institutions on the Internet, deregulation and liberalization of the financial sector, reducing barriers for mergers (Jimber, 2016).

The development of the world economic structure is necessarily accompanied by a cyclical shift in the instruments of capital accumulation (Glazyev, 2018). However, the issues of the economic content and the quantitative measurement of mergers and acquisitions remain insufficiently developed. As a result, despite the fact that quite a lot of studies of the dynamics, structure and geography of mergers and acquisitions have been published recently (for example, Ignatishin Yu.V., Gvardin S.V., Lugacheva S.I., Musatova M .M., Saratovsky A.D.), the problems of statistical recording of transactions of mergers and acquisitions are largely ignored.

The development of statistics of mergers and acquisitions in Russia will facilitate the involvement of the expert community in the production of statistics and the positioning of respondents as active professional users, which will compensate for the low degree of information disclosure on integration transactions and assess the integration activity of certain sectors of the Russian economy. The obtained results can serve as a justification for updating the production apparatus and restoring the structural integrity of the domestic economy.

## **1** Econometric approach to assessing the integration activity of the Russian sectors of economy

The analysis of the main trends in the development of the Russian economy testifies to the growing macroeconomic importance of economic integration. There is an active process of concentration of production and capital and today the Russian economy is the economy of large economic entities in the country (Karelina, 2015, Karelina, 2018).

Despite all the efforts of recent years, small and medium-sized businesses in Russia are not developed and occupy weak positions even in those industries in which small businesses traditionally dominate in the whole world. At the same time, medium-sized enterprises in Russia are organizational structures that are not structurally stable; they are the main targets for the absorption by large multifunctional holdings, which are based on the enterprises oriented to the raw materials sector.

There was a much greater volatility on the Russian M&A market for 2010-2016 in comparison with the world market. In 2016, the number of transactions in the Russian market increased by 2.34% and amounted to 481 transactions .The total amount of transactions in 2016 increased by 45.77% up to \$ 75.8 billion (see Figure 1), which is due to an increase in the share of larger transactions. The main contribution to the value of the Russian market of mergers and acquisitions in 2016 brought three transactions in the oil and gas sector:

- Sale of 19.5% of Rosneft shares for \$ 11.27 billion to a consortium led by the Qatari sovereign fund;
- Acquisition of a 49% stake in Indian company Essar Oil for \$ 6.33 billion by Rosneft;
- Acquisition of a 50% stake in Bashneft for \$ 5.3 billion by Rosneft.

# Fig. 1: Dynamics of the number and total value of M&A transactions in the Russian M&A market for 2010-2016



Source: author's own work

In 2016, the number of domestic transactions increased by 9.22% compared to 2015 (379 units), accounting for 78.79% of the number of transactions in the market of mergers and acquisitions. The cost of domestic transactions increased by 9.17% and amounted to \$ 39.3 billion (51.85% of the total cost of integration transactions in the Russian market of mergers and acquisitions in 2016).

Historically in Russia, the bulk of mergers and acquisitions have been carried out in the oil and gas sector and mining industry. However, in the process of implementation of the Government programs to reform the economy, there should be more incentives and opportunities for integration deals in a wider range of industries (Mkhitaryan, 2016). In this connection, there arises the need to analyze and formalize the tasks related to the comparison and classification of the branches of the Russian economy in terms of the level of integration activity.

Since the branches of the Russian economy are characterized by uneven flow of mergers and acquisitions processes, the main attention in the study of the spatial structure of the integration activity of Russian companies was paid to the indicator "The amount of mergers and acquisitions transactions", reflecting various aspects of integration activity in its integrated form. When determining the stratification structure, the information on the M&A activity of 16 branches of the Russian economy for 2014 and 2016 was used.

To make full use of the information that is contained in the values of this indicator, parametric classification methods were used (Arltova, 2015). Parametric modeling assumes the decomposition of the distribution law f (x). It is represented as a mixture k of distribution laws, each of which  $f_j(x, \theta_j)$  describes the distribution of a homogeneous j-th group of objects,

whose share in the total aggregation determines the weighting coefficient  $q_j \ge 0$   $(\sum_{j=1}^{k} q_j = 1)$  in the model:

$$f(\mathbf{x}) = \sum_{j=1}^{k} q_j f_j(\mathbf{x}; \boldsymbol{\theta}_j), \qquad (1)$$

where  $\theta_j$  is the vector of the law parameters of objects distribution of the j-q group.

To study the integration activity of the Russian economy, the method of splitting the mixtures of probability distributions was used, since each class is interpreted as a parametrically given single modal universal set with an unknown value of the vector parameter  $\theta_j$  defining it, and, accordingly, each of the classified observations is considered extracted from one of these general sets (Sirotin, 2016).

For an identical form of the distribution laws  $f_j(x, \theta_j)$  of each of the homogeneous groups, the problem of splitting the mixture of probability distributions can be represented in the form:

$$f(\mathbf{x}) = \sum_{j=1}^{k} q_j f(\mathbf{x}; \boldsymbol{\theta}_j)$$
(2)

The main problem here is the choice of the type of distribution law. The analyzed sign "The sum of mergers and acquisitions" is the result of the combined effect of many factors, among which, due to a sufficient variety of elements of the economic activity of each sector of the Russian economy, there are clearly no dominant, and the nature of the action of each factor, observable as well as latent, on the resultant value of the indicator can be considered multiplicative. Therefore, we can assume that in this sense for a homogeneous group of the Russian economic sectors, the law of distribution of the feature will be logarithmically normal:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}},$$
 (3)

where  $\mu$  and  $\sigma$  are, respectively, the mathematical expectation and the standard deviation of the value lnx.

It is expedient to estimate the vector of model parameters  $\theta = (q_1, \dots, q_k; \theta_1^T, \dots, \theta_k^T)^T$  by the maximum likelihood method (Dubrova, 2003) by maximizing the logarithm of the likelihood function:

$$\ln l(\hat{\theta}_{omr}) = \max_{\theta} \sum_{j=1}^{k} \ln f(x;\theta)$$
(4)

After receiving the estimates, the question of classifying the objects arises. To obtain an unambiguous rule for assigning an arbitrary object to one of the allocated classes, one can use the Bayesian approach, which uses as a criterion the minimum of the average risk of the erroneous classification:

$$\bar{r} = \sum_{i=1}^{n} \sum_{j=1}^{k} r(i, j) P(i, j) = \sum_{i=1}^{n} \sum_{j=1}^{k} r(i, j) P(i / j) P(j) = \min$$
(5)

The decision rule depends on the chosen function of the cost r(i, j) of the transfer of the i-th object to the j-th class. With a simple cost function  $r(i, j) = -\delta_{\eta}$ , where  $\delta_{\eta} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$  is the Kronecker symbol, the condition of correct classification will be determined by the expression:

$$-\sum_{i=1}^{n} P(i/i)P(i) = \min \quad \text{or} \quad \sum_{i=1}^{n} P(i/i)P(i) = \max .$$
(6)

The probability of a correct classification over the interval of the width  $\Lambda x$  in the neighborhood of the point x can be represented by an approximate equation that relates the probability P (i / i) of the correct assignment of an object to the i-th class and the probability density of the i-th component of the mixture of distribution laws. Using this connection, it is possible to proceed to the classification rule:

$$\sum_{i=1}^{n} P(i)f_i(x) = \max.$$
(7)

The value of P (i) is the a priori probability of assigning an object to the i-th class, equal to the fraction of objects of the i-th class in the total aggregate. Its empirical analogue is the estimation of the weight coefficient  $q_i$ :  $P(i) = q_i$ . This estimate can be used in the final decision rule:

$$\sum_{i=1}^{n} \overline{f_i}(x) \overline{q_i} = \max, \qquad (8)$$

which determines the boundary between neighboring groups as the abscissa of the intersection point of their weighted model probability densities.

Analysis of the histogram of the sum logarithm of M&A transactions for 2014 and 2016 showed the presence of three strata with bell-shaped distribution functions. The average value  $\mu_j$  for the logarithm of each stratum was defined as the abscissa of the point of its assumed maximum, the standard deviation of the sign logarithm was determined by the distance along the abscissas axes from the assumed point of the maximum to the inflection point of the probability density of the sign logarithm; the weight coefficient of each stratum is proportional to the area under the assumed weighted curve of its distribution in the mixture (Table 1).

Tab. 1: Initial parameters approximations of the distributions mixture, 2016

Stratum number	μ theor.	$\sigma$ theor.	q theor.
1	-2.35	1.43	0.4
2	0.29	1.08	0.3
3	1.87	0.62	0.2

Source: author's own work

Obtaining the best values of the parameters requires the selection of an optimality criterion and the organization of the procedure for obtaining estimates. For this, as it was mentioned above, the maximum likelihood method was chosen. The obtained parameter estimates for 2014 and 2016 are presented in Table 2.

Stratum	2014			2016		
number	μ theor.	$\sigma$ theor.	q theor.	μ theor.	$\sigma$ theor.	q theor.
1	0.89	0.42	0.72	-2.85	0.72	0.21
2	-1.72	0.38	0.14	0.02	0.81	0.45
3	1.81	0.04	0.19	1.74	0.28	0.35

#### Tab. 2: The obtained parameter estimates

Source: author's own work

The parametric model well describes the available data on the integration activity of the sectors of the Russian economy, as evidenced by the proximity of theoretical and empirical histograms. The deviation of the theoretical from the empirical distribution of the sum of transactions of mergers and acquisitions of branches of the Russian economy was  $\xi = \sum \frac{|y_t - y_e|}{n} = 0.012$  in 2014, and  $\xi = 0.014$  in 2016.

By using the criterion of minimum mean risk of classification errors (Ayvazyan, 2001), the stratum boundaries were found as the abscissas of the intersection points of weighted distribution curves for adjacent strata. Results shows that the number of branches of the Russian economy with low integration activity did not change for the period from 2014 to 2016 and amounted to 12.5% of the total number of researched industries (hotel business, leisure, tourism and mass media). In 2016, compared to 2014, the number of branches of the Russian economy with average integration activity increased from 62.5 to 68.75%, and the number of industries with a high level of integration activity decreased from 25 to 18.75% of the total number of researched industries. The composition of the leading third stratum is presented in Table 3.

Number of object	2014	2016	
1	Oil and gas industry	Oil and gas industry	
2 Metallurgy and Mining		Metallurgy and Mining	
3	Chemical industry	Chemical industry	
4	Banking and insurance services		

Tab. 3: The composition of the third (leading) stratum for 2014 and 2016

Source: author's own work

The largest deal in the metals and mining industry was the sale of 32% of Polyus Gold shares by minority shareholders for \$ 3.74 billion in 2016, while in the chemical industry the ONEXIM Group sold 20% of Uralkali shares for 1, 65 billion US dollars.

The oil and gas industry is of interest to international and national oil and gas companies due to relatively low technological risks, capital and operating costs and the cost of assets. Due to a sharp reduction in financing for geological exploration, caused by a fall in oil prices from mid-2014, 2016 was the worst year in terms of discovering new deposits over the past 70 years.

The branch of banking services and insurance in 2016 compared with 2014 moved from the stratum with high integration activity into the stratum with average integration activity. In 2016, the value of the market for mergers and acquisitions in the banking services and insurance sector was 28.95% of the M&A market value in 2014 (\$ 1.1 billion), and the quantitative volume of the M&A market was 74, 58% of the quantitative volume of the market of mergers and acquisitions in 2014 (44 transactions).

This is due to the fact that buyers are trying to get a significant discount to the value of net wealth in order to compensate for inherited risks, while sellers insist on the maximum price, with some willing to wait for an increase in value as the economy and exchange rates recover.

Russia's desire to increase the level of supply of domestic foodstuffs, the ban on the import of food products from Europe and the USA and the localization program led to an increase in production in the agricultural sector. As a result, the agricultural sector moved from the stratum with low integration activity into the stratum with average integration activity in 2016, compared with 2014.

At the end of 2016, the value of the M&A market in the agricultural sector was 150% of the M&A market value in 2014 (\$ 1.5 billion), while the quantitative M&A market volume was 124.24% of the quantitative volume market of mergers and acquisitions in 2014 (41 transactions). It should be noted the activity of investors from the countries of Asia and the Middle East.

Thus, the use of distributions decomposition in the integration activity modeling of the Russian economy sectors made it possible to identify 3 groups of industries with different levels of integration activity. After the analyses of the M&A transactions, on 16 branches of the Russian economy for 2014 and 2016, it is possible to draw the following conclusions:

economic sanctions did not entail a massive redistribution of property in Russia;

• outsider industries and a number of specific companies-leaders (NK Rosneft) had been indicated, including due to the number of transactions and the value of the M&A market.

## **2** Conclusion

The proposed statistical approach to assessing the integration activity of various sectors of the Russian economy in the framework of development of domestic M&A statistics, based on the use of splitting algorithms for probability distribution mixtures, made it possible to quantify sectoral disproportions and build a statistical classification for identifying integration-active sectors of the Russian economy.

On the basis of the analysis presented, it is possible to elaborate measures of state policy in order to improve the efficiency of the integration activity of various sectors of the Russian economy. The obtained results can serve as a justification for renewal of the production apparatus and restoration of the structural (branch) integrity of industry, which leads to acceleration of innovation and technological development and the formation of unique areas of the reindustrialization of the domestic economy.

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### Contact

Mariia Karelina Nosov Magnitogorsk State Technical University 455000, Russian Federation, Magnitogorsk, Lenina st., 38 marjyshka@mail.ru Tatiana Ivanova Nosov Magnitogorsk State Technical University 455000, Russian Federation, Magnitogorsk, Lenina st., 38 jun275@mail.ru Violetta Trofimova Nosov Magnitogorsk State Technical University 455000, Russian Federation, Magnitogorsk, Lenina st., 38 violat@mail.ru