CALCULATION OF THE NORMAL LENGTH OF LIFE USING THE WEIBULL MODEL

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Abstract

In the latest years, there is an intensive discussion about the increase in the length of human life. People live till higher age, but this is not, on the other hand, supported by sufficient natality. That is why required reforms are being more and more discussed.

From a demographic point of view, it is important to have the most accurate imagination about till what age people will most often live. This is also related to the calculation of the normal length of life, which is the age at which most of people die.

In this article, a derived estimating formula obtained by using the Weibull model will be used for calculating the normal length of life. For comparison, an approximate estimation of the normal length of life will be used.

The aim of this article is to show the possibility of the estimation using the derived function obtained from the Weibull model. Then, compare the obtained results with approximate estimates of normal length of life.

Key words: mortality, normal length of life, Weibull model, the Czech Republic

JEL Code: J10, J11, J19

Introduction

In recent years, mortality has improved (Boleslawski and Tabeau, 2001 or Koschin, 1999). It is decreasing during the whole human life. It also means that more people live till the highest ages (Langramrová JI, Fiala, 2013). Because of low natality aging of population occurs. One solution may be to increase the retirement age (Fiala, 2013). But it is also necessary to focus the attention on the care of the elderly.

That is the reason why we need to focus on the most accurate capture of mortality (Gavrilov, Gavrilova, 2011 or Kannisto et al., 1994).

The most often used indicator is life expectancy (Burcin et al., 2010). However, this characteristic has its drawbacks. Therefore, it is good to use a normal length of life as another characteristic. It very important to compare these two characteristic (Dotlačilová, 2014).

1 Methodology

Several indicators can be used for the analyzing of mortality. One of the most often used is the life expectancy. Another one can be the normal length of life – modal length of life (Langhamrová, JA, Arltová, 2014).

Life expectancy is a type of an average (Fiala, 2002). So it has its disadvantages - it can be affected by an extreme values. In the past, it was influenced for example by high infant mortality. Therefore, it is good to use the other length of as an another indicator.

The normal length of life is the age at which most people die. It's the modal age of deaths. Its advantage is that as a modal characteristic it is not affected by extreme values. Various methods can be used to calculate it. One of the options is an approximate estimate. We calculate the age obtained as the age at which the number of deaths is maximal. The second option is to use the analytical functions. Here it is necessary to find the maximum of the density of the deaths.

1.1 Use of Weibull model

Normal length of life can be estimated like an age in which the density of death has got its maximum.

For own calculations we can consider $x > 60$. Probably most people die in higher age than 60.

For deriving the estimating formula we have to introduce the density of death (Koschin, 2000):

$$\delta(x) = l(x) \cdot \mu(x) .$$

Function of $l(x)$ (for $x > 60$) could be written like:

$$l(x) = l(60) \cdot e^{-\int_{60}^{x} (bt^a) dt} .$$

Instead of $\mu(x)$ will be substitute the Weibull model.

Now we can use the condition for the existence of a local extreme: $\delta'(x) = 0$,

where $\delta(x)$ is the density of death.

For finding the estimating formula which could be used for calculating of the normal length of life from the Weibull model, we use following condition (Fiala, 2002):

$$\mu'(x) - \mu^2(x) = 0, \tag{1}$$

where $\mu(x)$ is the intensity of mortality, which is modelled by Weibull model (Thatcher et al., 1998, Dotlačilová, 2019):

$$\mu(x) = b \cdot x^a. \tag{2}$$

The first derivative of the intensity of mortality: $\mu'(x) = a \cdot b \cdot x^{a-1}$.

The second power of the intensity of mortality: $\mu^2(x) = b^2 \cdot x^{2a}$.

The last step is about expression x of from:

$$a \cdot b \cdot x^{a-1} = b^2 \cdot x^{2a}$$

Final estimating formula for normal length of life:

$$x = \left(\frac{a}{b}\right)^{\frac{1}{a+1}}, \tag{3}$$

where a and b are estimated parameters from Weibull model.

1.2 The estimation of the normal length of life

During the estimating of the normal length of life we will come out of its definition. Normal length of life is the age at which most people die. This is the modal age of death.

At first, we find the age at which the number of deaths (from mortality table) is the highest. The found age is further corrected by adding a value of 0.5 (we add a half of the age interval).

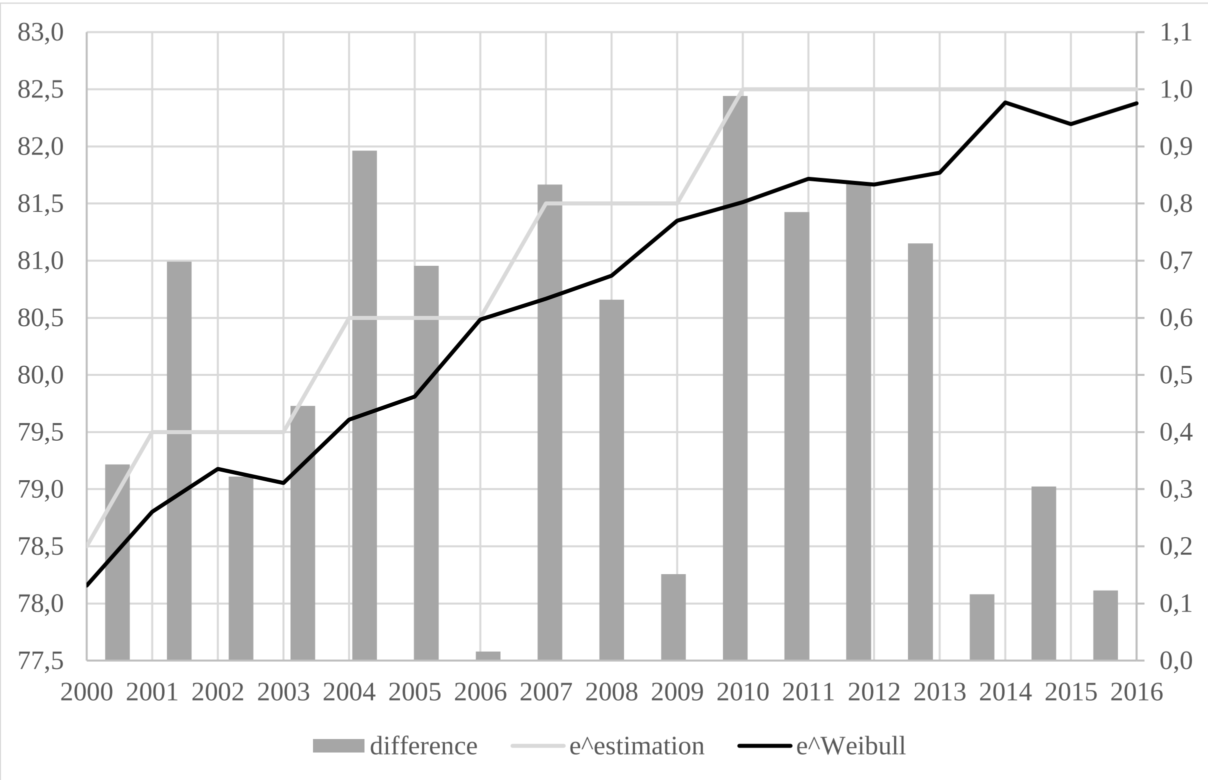
2 Results

During the estimating of the normal length of life we will come out of its definition. Normal length of life

Data about mortality for Czech population were used for calculation of the normal length of life (year 2000 was the initial one). The aim is to show and compare the possibilities of estimating the normal length of life using the Weibull model. Both graphs show the results obtained by using the Weibull model (e^Weibull) and their approximate estimates (e^estimation). The outputs are also supplemented by displaying the differences between these two methods.

The first figure shows the values of normal length of life for males in the Czech Republic. While the normal length of obtained from derived formula shows relatively smooth development, the other one shows significant changes. If we look at the differences between these two methods, we find that the highest difference is 0.99 (year 2010). Conversely, the lowest difference is 0.02 (year 2006) (all values could be find im the Attatchement 1). It is also not possible to observe any trend from the differences obtained. It is only possible to observe that the values of the differences have decreased in the last three years.

Fig. 1: Normal length of life - Czech Republic, males

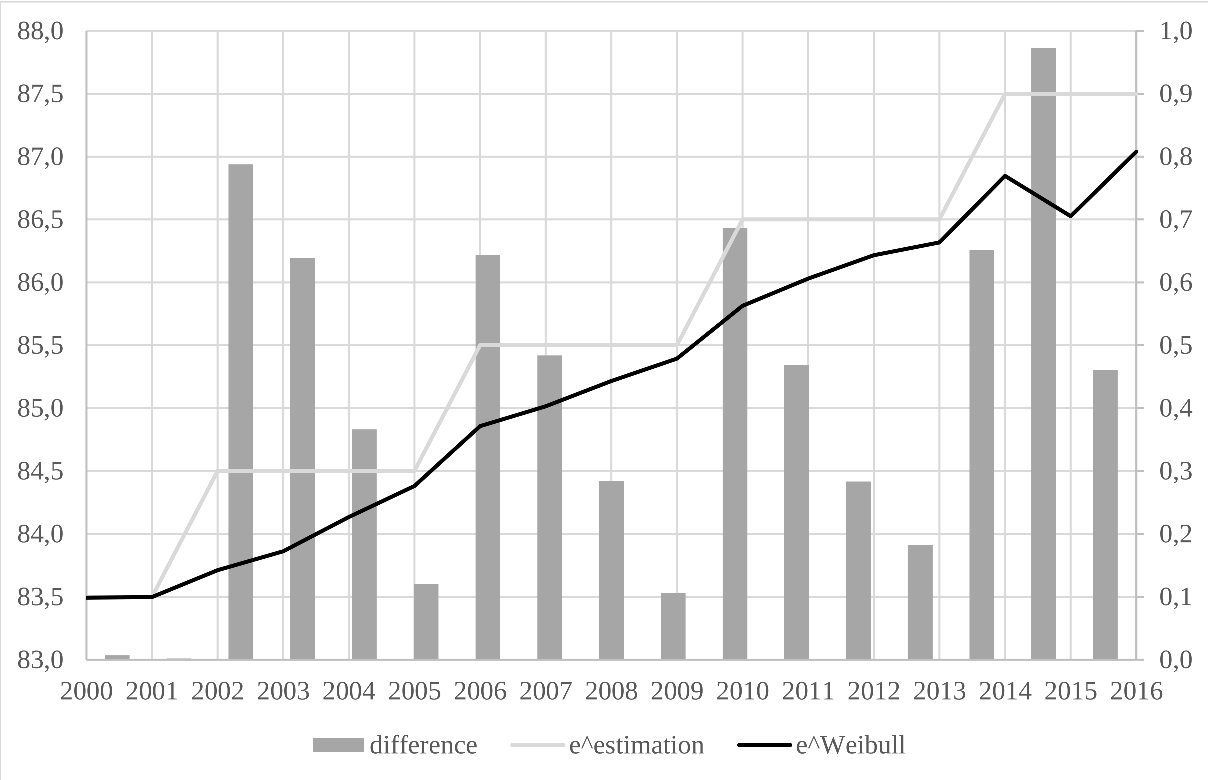


Source: data – Eurostat (2018), author´s calculations

The second figure shows the calculated values of normal length of life for females in the Czech Republic.

As in males‘ population, the derived formula from the Weibull model shows a smoother development than an approximative one. For better comparison of both methods there are values of differences displayed in this graph. The highest difference is 0.97 (year 2015) and the lowest is 0.002 (in 2001) (all values could be find in the Attatchement 2). The normal length of life shows an upward trend (with the exception of 2015, when there was a slight decline). But this cannot be deduced from the differences obtained.

Fig. 2: Normal length of life - Czech Republic, females



Source: data – Eurostat (2018), author´s calculations

Conclusion

The goal of the paper was to show the possibilities of calculating normal length of life using the Weibull model or approximate estimate and their subsequent comparison.

Data about mortality of males and females in the Czech Republic were used for the calculations.

During comparison of these two methods used in males‘ population it is clear that in both cases the normal length of life is increasing. For derived formula, the increase is relatively smooth (as opposed to an approximate estimate). It is also interesting the comparison of the differences obtained. While values of the normal length of life gradually increase, for differences no trend can be observed.

It is also true for Czech females. The normal length of life has increased (which is in line with the positive development of mortality). Here again, it is clear that the approximate estimate provides step changes in normal length of life. In contrast, values obtained from a derived formula provide smoother development. If we focus on the differences between these two methods, we can not observe any trend. Moreover, in contrast to the males‘ population, an increase in differences at the end of the reference period can be observed.

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Attachment 1: Values of the normal length of life, differences – Czech Republic, males

	2000	2001	2002	2003	2004	2005	2006	2007	2008
e^estimation	78,5	79,5	79,5	79,5	80,5	80,5	80,5	81,5	81,5
e^Weibull	78,2	78,8	79,2	79,1	79,6	79,8	80,5	80,7	80,9
difference	0,34	0,70	0,32	0,45	0,89	0,69	0,02	0,83	0,63

	2009	2010	2011	2012	2013	2014	2015	2016
e^estimation	81,5	82,5	82,5	82,5	82,5	82,5	82,5	82,5
e^Weibull	81,3	81,5	81,7	81,7	81,8	82,4	82,2	82,4
difference	0,15	0,99	0,78	0,83	0,73	0,12	0,30	0,12

Source: data – Human Mortality Database, author´s calculations

Attachment 2: Values of the normal length of life, differences – Czech Republic, females

	2000	2001	2002	2003	2004	2005	2006	2007	2008
e^estimation	83,5	83,5	84,5	84,5	84,5	84,5	85,5	85,5	85,5
e^Weibull	83,5	83,5	83,7	83,9	84,1	84,4	84,9	85,0	85,2
difference	0,01	0,00	0,79	0,64	0,37	0,12	0,64	0,48	0,28
	2009	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	
e^estimation	85,5	86,5	86,5	86,5	86,5	87,5	87,5	87,5	
e^Weibull	85,4	85,8	86,0	86,2	86,3	86,8	86,5	87,0	
difference	0,11	0,69	0,47	0,28	0,18	0,65	0,97	0,46	

Source: data – Human Mortality Database, author´s calculations

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