# ŠIMERKA'S SUBJECTIVE INTERPRETATION OF PROBABILITY THEORY

Jan Coufal – Petra Dotlačilová – Jiří Tobíšek

#### Abstract

The text is dedicated to Václav Šimerka's book The Power of Conviction from the 1880s, in which a long time before creation of the subjective probability in the 1930s. show subjective probability FP Ramsey and B. de Finetti, who laid the foundations of subjective interpretation of probability independently of each other. Not only life is mentioned in the article (Šimerka worked as a chaplain, then as a substitute secondary school professor of mathematics, physics and Czech, later as a priest), but also in brief Šimerkův view of subjective probability in his pivotal work The Power of Conviction, which is in two language versions: Czech and extended German. He wanted to calculate the power of conviction and give it a number. "Conviction" refers to "a number of degrees of knowledge" - from "empty mind" (beliefs with zero value) to suspicion, hypothesis, hypothesis to necessary knowledge (beliefs of 1); it depends on the probability ("fidelity") of the reasoning of the courts. German scholars (eg Carl Theodor Michaëlis) gave a negative assessment of the text (which probably led to his being almost unknown in the world), but Tomáš Garrigue Masaryk highly appreciated it on the Czech side.

**Key words:** probability, subjective interpretation of probability, Šimerka, Masaryk, power of conviction

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## Introduction

Humans have met with random phenomena since their distant past, but their mathematical research has only taken place in the new age. The causes of this relatively late start of the mathematical approach could be several:

- a) No one has discovered the relationship between accurate mathematics on one hand and random phenomena on the other.
- b) Mathematical research of random phenomena was not necessary before, because no one needed it. The need for a precise description of random phenomena has dicovered not only

in connection with development shipping, business and with the development of state (the reason for emergence of demography and insurance), but also in connection with the development of natural sciences (especially astronomy and experimental physics) and which means need for measurement of results loaded by random errors.

In practice, we often encounter so-called subjective probability. It is an inherent (subjective) estimate of objective probability. It turns out that these estimates tend to be quite distorted, especially either by the optimistic or pessimistic approach of the evaluator (subject). Furthermore, the resulting probability is distorted by the subconscious effort to symmetrize the distribution. Another problem is the frequent overestimation of probability unlikely phenomena and vice versa non-appreciation of the probability of a phenomenon with high probability. It can be expressed verbally and numerically. Verbal expression of probability is often more comprehensible, but it cannot be used to create mathematical models. There is no norm for an unambiguous relationship between verbal and numerical expression. Therefore, people can understand the expression differently and can give them different meanings. Subjective probability is the probability we assign to the result of an experiment that is not repeatable under the same conditions, eg our hockey players will win the next hockey game with Russia, this year in July there will be no floods in Moravia. An important feature of subjective probability is the fact that its value is usually very important for decision-making purposes and for solving serious problems. The numerical expression of probability is usually determined by the analyst in collaboration with an expert in the field.

It is usually stated, the first subjective interpretation of probability is given by F. P. Ramsay (1931)<sup>1</sup>. Independently on Ramsay, B. de Finetti (1937, 1974, 2006) also dealt with the same problematic. Statistician de Finetti has been developing fro the whole life powerful mathematical-psychological methods. It's incredible, but he also discovered a way to objectively measure subjective probability. The truly influential development of the subjective interpretation of probability was the books of L. J. Savage (1954), and the articles Mellor (1998), Rossi (2001) and Winkler (1996).

Šimerka (1882) begins her writings: "When the famous philosopher Herbart found that calculation in the psychology takes of the place, without the deed that can be regarded as mere desire, nobody blame me, I guess nobody, if I introduce them into the logic or into the methodology and also I will introduce the metaphysics and thus I will explain many things both in scientific research and in history. After all, the mathematics is perfectly intelligible,

<sup>&</sup>lt;sup>1</sup> The article was written in 1926 and published in 1931.

and in other sciences used not only qualitative but quantitative results, ie it shows what the subject is and how great it is."

It is actually a treatise on what is now called subjective probability. Thus, Šimerka becomes the forerunner of the subjectives probability and the first mathematician in the Czech Republic to deal with applications of mathematics in psychology. In this paper, there will be not about the inclusion of Šimerka among the founders of mathematical psychology, but it will be the inclusion of Šimerka among the founders of the subjective probability theory. Ramsey and B. de Finetti developed the theory of subjective interpretation of probability independetly each other - and also unfortunately independetly on Václav Šimerka.

# 1 Václav Šimerka

According to Panek (1888) and Hykšová (2006), 200 years ago, Václav Šimerka (more precisely on 20th December 1819) was born in Vysoká Veselí to a family of coopers and later a businessman with a yarn of Petr Šimerka and his wife Terezie. Šimerka received his basic education at the parish school in Vysoké Veselí, then graduated from the grammar school in nearby Jičín with excellent learning outcomes, where taught professor of mathematics Franz Mühlwenzel (1793–1858) and philologist Simon Karel Macháček (1800–1846).

In the years 1839/40 and 1840/41 Šimerka studied at the Faculty of Philosophy of the University of Prague, where he graduated again with obligatory teaching of religion, philosophy, mathematics, Latin philology, natural science, physics, moral philosophy and history and also lectures and exams of mathematics under Jakub Filip Kulik (1793–1863) and from astronomy and practical geometry under Adam Bittner (1777–1844). He continued his studies at the Theological Seminary in Hradec Králové, where he was ordained a priest on 25th July 1845.

He then worked as a chaplain in Žlunice near Vysoké Veselí. Five years later, after the so-called Exner-Bonitz reform of education, when the faculties of philosophy gained equal status with other faculties and state examinations were introduced for secondary school teachers, Šimerka apparently due to disagreements with the parson from Žlunice applied for a state examination in mathematics for teaching at lower and higher grammar schools and physics for teaching at lower secondary schools. Teaching proficiency exams consisted of four parts: homework, clausure work, oral exam and lectures. In April 1851, Šimerka successfully completed the math exam and obtained a certificate for both grammar schools in Czech and in German. However, the physics exam was inadequately evaluated and Šimerk

was advised to complete his education in physics; he could then undergo the new test after 18 months. He remained in Žlunice until Christmas in 1851; until Easter of the following year he was a personal chaplain in nearby Slatiny.

He then went to Prague and in the summer semester of the school year 1851/52 he enrolled as a regular student at the newly organized philosophical faculty, where he also studied in the following two semesters. He completed lectures in optics, statics, electricity and magnetism, thermodynamics, experimental physics focused on teaching, astronomy and chemistry.

On 17th February 1853 Šimerka applied for a new state exam in physics, this time for lower and higher grades of grammar schools. While in his first attempt his written works were considered to be totally inadequate and the oral examination was considerably unfavorable, the result was now vastly different. For example, in the appraisal of homework, we find a statement that demonstrates Šimerka's extraordinary diligence, strict logical thinking, a special gift of clear expression and expertise. Great praise was also given to the final work and final oral exam, which took place on 27th June 1853. According to Petřina's final report, the examining board decided, based on her previous performance, that the candidate had received a degree in physics for the whole grammar school. But also in every respect this excellent candidate deserves a recommendation to actually teach. In doing so, he can use German as well as Czech.

In the autumn of the same year, by the decree of the Ministry of Culture and Teaching of 29th October 1853, Šimerka was named as a substitute teacher at the Piarist Grammar School in České Budějovice. In the first year of his pedagogic activity he taught physics and Czech language, and in the following years he taught Czech language and mathematics. At that time, Šimerkova's scientific treatises (Šimerka, 1858, 1858a, 1859) were published in the Sitzungsberichte of the Vienna Academy of Sciences. However, even after nine years, Šimerka did not wait the tenure.

Therefore, at the end of the school year of 1861/62, he returned to spiritual administration at his own request. On 20th July 1862, he was appointed pastor in Slatina near Žamberk, less than four years later (12th June 1866) went to Jenišovice near Vysoké Mýto, where he spent twenty years as a pastor. Šimerka did not return to the teaching profession, but he was still interested in mathematics. Šimerka was one of the founders of the Union of Czech mathematicians and physicists. In 1886 Šimerka retired; he spent the last months of his life in village Praskačka, where he died on 26th December 1887 and where he was buried.

### **2 Power of Conviction**

In 1882, Šimerka's treatise "The Power of Belief" was published, where he attempted to create a theory that would use quantitative probabilities to quantify degree of conviction. The aim of his article was expressed in words (Šimerka, 1882, pp. 75-76): "As white, gray, black, red, etc., it could be included in the word color, so the concepts can be: hunch, assumption, possibility, probability, hypothesis, faith, knowledge, certainty and so on. being summarized in one, that is the belief or conviction."

Causes or sources of conviction Šimerka calls the reasons their power and fidelity. In addition to other examples, he recalls the various situations in practicewhere reasons are fairly appreciated: life insurance, where calculations are based on mortality tables, insurance against fire or hail, based on statistical data, or health assessment about individuals for a given year expressed by the number of deceased per 1000 inhabitants. He adds, however, that it is not always possible to specifically determine the power of conviction.

Simerka asks: how can the conviction be expressed by numbers? He states (Simerka, 1883, p. 517): "For this purpose the probability calculus is exceptionally convenient, since our conviction about the possibility of an event increases in the same rate as does the mathematical probability, that is, everything is more believable, the more it seems to be probable. The terms in the sequence [...] empty mind, feeling, ..., up to knowledge and certainty can therefore be expressed by numbers between 0 and 1, where 0 corresponds to none, 1 to the highest conviction." Causes or sources of the conviction are called grounds, their power v is expressed by probability. To assemblemore convictions together, Šimerka introduces the concept of an imperfection of a conviction as a difference  $\varepsilon = 1 - v$  between the complete knowledge and the given conviction v. Consider convictions  $v, v', v'', \ldots$  and the corresponding imperfections. The resulting power of conviction V is given by the formula  $1 - V = (1 - v)(1 - v')(1 - v'') \ldots$ , which can be expressed as follows: the imperfection of a human conviction is a product of imperfections of its grounds.

For  $v = v' = v'' = \dots = 0$  we have V = 0; according to Šimerka's words: *empty* grounds provide no belief. For  $v' = v'' = \dots = 0$  we obtain V = v and the characterization: *in* an empty mind every ground enroots with its full power. Šimerka (1883, p. 517) continues: "This is attested not only by the experience from schools and common men, many of which believe even very shaky novels and stories, but also the experiences of missionaries who give evidence that Christianity enroots the best in the nations with disordered minds, when their original superstitions were rebuttet, without being substituted by anything else; otherwise is it

much more difficult. [...] The empty mind can therefore be deceived by false grounds, what would be otherwise not so simple. It is clear that this is the basis of the old immoral principle: slander, something will stick in the memory."

If v = 1, it comes out for any positive and negative values v', v'', ... result V = 1, or (Šimerka, 1882, p. 97) "right (objective) conviction can not be increased by any new reasons, nor by the anti reasons decreased. According to the mathematical way of speaking we can therefore call the force of truth infinitely great; since the final variables disappear against it."

In the other Šimerka considers *n* reasons for "strength" *v*, ie.  $1 - V = (1 - v)^n$ . E.g. for  $v = \frac{1}{2}$  and n = 10,  $V = 1 - \left(\frac{1}{2}\right)^{10} = 0,99902347$ . Next, Šimerka asks the question of perfect knowledge. He notes that mathematicians are generally satisfied with the 7.3 arithmetic degree of conviction. He adds that astronomy and measurment are usually satisfied with the same precision, where the degree corresponds to a  $\frac{1}{20}$  mm error per kilometer. Similarly, if, for example, a weight of 1 pound due to the weight of the whole Earth is neglected, this error corresponds to a 24.9 degree of conviction. In total, Šimerka (1882, p. 94) concludes: "When all mathematicians with a higher degree of 7.3 are so satisfied that many of them call the results of their numbers infallible, I believe that the 25-degree conviction can always pay for perfect."

At the same time, Šimerka answers the problem of induction. He notes that every inductive reason, that is, a reason based on experience or observation, it has validity less than 1, but there is a greater amount, so it is possible to arrive at a certainty (Šimerka, 1882, p. 94-95): "Our belief in the reality of the external world, for example, rests on our eyesight and on our feel, then we do not make a mistake if we give them three degrees of strength; surely both of us do not deceive us together in 1000 cases once. Then the threefold experience has more power than the calculating certainty, 9<sup>tera</sup> equals total knowledge, not including other people's testimonies, analogies with similar things, thoughts risen from feelings and impressions of other senses. The same is true for the other types of induction."

Šimerka concludes her article with words (Šimerka, 1882, p. 111): The subject itself, from which I am only a mathematical amount, namely the rise and controversy of belief was talked out, leaving the spread (scientific research) and communicating the beliefs of the future, is as serious as barely another part of calculation. Do not act here any more or less than the power of truth. And who can deny her mightiness? To act not only in private conversations, in schools, writings, and on the rhetoric, but it is also armed the arms, shed blood on battlefields, not even afraid of death on the gallows, knowing that the body may be frustrated, but not spirit. [...] In addition, materialism that has seized our century cannot be on detriment, but it will also count on something spiritual. For these and similar reasons, I hope that I will not remain a lone worker in this new field.

#### Conclusion

The theory to which Šimerka directed was systematically formed only in the 1930s. It is obvious that just as the level of conviction, the probabilities were understood by the representatives of the subjective interpretation of probability. The Czech version of Šimerk's treatise (1882) was briefly mentioned by František Josef Studnička in the reference journal Jahrbuch über die Fortschritte der Mathematik for the year 1882: *This treatise, also published in the Sitzungsberichte* [...], wants to be considered a "mathematical-philosophical attempt" to justify and numerical expression of different degrees of conviction, analogous with probability.

In the following year, the German version (Šimerka, 1883) was very unflatteringly evaluated by the German philosopher Carl Theodor Michaëlis (1852–1913): *Šimerka believes he can measure convictions. What matters to him is the expression: objective conviction.* "This is said to arise from external (real) causes." [...] The author pays tribute to every sentence "Every calculation is better than no calculation." What is the basic measure to be measured [...] author could determine inadequately, as from the beginning, the dead mathematical psychology of Herbart, to which he refers in the introduction, was able to give a basic measure of imagination as a force.

As was mentioned earlier, the quoted claim that each calculation is better than no calculation has been referred to by Šimerka as a long time ago established principle, he said only in a note following a more convincing justification for the applicability of the probability count for the given purpose. As for the objective conviction, after the sentence quoted in review of Šimerka (1883, pp. 515–516) continues: *Many hypotheses, then knowledges and cognitions, belong to the objective conviction; to the subjective again, the suspicions and assumptions.* [...] *The most beautiful aspects of modern science were, in the minds of scholars, only a hint of suspicions and assumptions, and they came to the public and became hypotheses until they finally acquired the validity of suspicions and assumptions. The writer traced the same march to these considerations; and therefore he believes that they must undergo many complications before the equal right among other sciences is given to them.* 

Sadly, Šimerk's words were fulfilled and his work, although published in Vienna in German, remained unnoticed for a long time. Today we can only speculate whether the situation would change if Michaëlis, who wrote review and critisized the philosophy based on empirical psychology, and so he may not even seek a deeper understanding of Simera's work. Let's add that the "dead" herbartism was the official philosophy at the Austrian schools. Professor of Philosophy at the Prague University Josef Dastich (1835–1870) and his successor Josef Durdík (1837-1902) were among the orthodox herbartists. The greatest acknowledgment of the quoted Šimerek's writings was the mention in Masaryk's Logic Review (1884, p. 137), where Šimerka's (1882) is referred to as a brilliant publication. In Masaryk (1885) there are words of appreciation for Šimerka. More than 50 years later, Šimerkův's contribution was highlighted by Nejedlý (1937, pp. 165-166): [...] what Šimerka did here was one of the greatest acts of modern mathematical philosophy: he interpreted noetics, the credibility of knowledge, on the base of mathematics. [...] Masaryk was also captivated by the publication. And he also had a different interest: Šimerka also solved the probability problem, which Masaryk was very interested in at the time, and so there was a personal introduction. "Honored Friend," writes Masaryk Šimerkovi II. 84, which shows that they were good friends at that time. I also write to him: "Please, write me the most important deviations of Your from the ordinary theory of probability, such as Laplace, I want to write about your publication into Athenaea and the German philosophical journal write. In doing so I will point out what the greatest importance has for philosophy. Also a divergent opinion in some things. I will then send this to you first when I have the time to study your book more thoroughly one more time [...] And so we can see how Masaryk was able to go beyond the boundaries of university only learning and society. On the contrary, this country priest was a nicer and bigger scholar to him than anyone else.

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## Contact

Jan Coufal

University of Economics and Management, Nárožní 2600/9a, 158 00 Praha 5, Czech Republic jan.n.coufal@seznam.cz

Petra Dotlačilová

Faculty of Informatics and Statistics (Dep. of Mathematics)

University of Economics, W. Churchill Sq. 3, 130 67 Prague 3, Czech Republic

petra.dotlacilova@vse.cz

Jiří Tobíšek

University of Economics and Management, Nárožní 2600/9a, 158 00 Praha 5, Czech Republic jiri.tobisek@vsem.cz