AN ACTUARIAL APPROACH FOR CREDIBILITY MODELLING FOR EXTREME CLAIMS APPLIED ON NON-LIFE CZECH INSURANCE MARKET

Karina Benetti

Abstract

Credibility theory is an actuarial approach that is used to calculate insurance premiums. This article aims to estimate the credibility premium based on actual data from the non-life Czech insurance industry for 13 years (2006 to 2018), considering the amount of incurred claims from three insurance branches and the number of extreme losses in each risk area. The analysis was based on Bühlmann and Bühlmann Straub credibility models' assumptions to estimate the net credible premium for the upcoming year as a linear function of the prior claims and the number of extreme events. This multivariate model allows estimating the conditional mean square error of prediction for the credibility predictor of the ultimate claim. Furthermore, considering the number of extreme losses, the results of this paper can be a good guidance to the Czech insurance industry in the case of occurring extreme losses in natural hazards. In the discussion of the article, the estimated values will be compared with the real values.

Key words: credibility theory, credibility modelling, extreme claims.

JEL Code: C13, C52, C58

Introduction

Credibility theory is an actuarial approach that is used to calculate insurance premiums. Credibility theory has been discovered and derived during the 18th century by Bayes (1763). Moreover, Bühlmann (1967) and Bühlmann-Straub (1970) introduced a multivariate generalization of the credibility model for claim reserving.

The Czech Republic faces high losses caused by natural storms and floods over recent years; the Czech insurance market has experienced extreme losses of natural hazards that would affect the calculation of premiums to cover the expected liabilities in the occurrence of such risks. Since credibility theory provides an actuarial approach to deal with these extreme losses, it will be important to explore the implementation of credibility models in such risks.

Bühlmann's approach predicts net premium based on the prior information for insurer's claims as a linear predictor assuming that data are identically distributed. This method is called the greatest accuracy approach. Although, on the other hand, Bühlmann-Straub approach is an extent of Bühlmann credibility where data are not identically distributed, this method overcomes the limitation of Bühlmann approach.

This paper uses a modified model based on the Bühlmann and Bühlmann-Straub models to project the premiums based on the past /posterior data of extreme losses by natural hazards collected from the Czech insurance market from 2006 to 2018.

1 Method of Research

1.1 Review of the Literature for the Last Decade

Linda and Kubanová (2012) used actual data from five insurance companies to calculate premiums for motor third-party liability insurance based on Bühlmann-Straub credibility as a methodology to improve the quality of net premium estimation.

Loisel and Trufin (2013) considered the discrete-time ruin model to determine the characteristics of the ruin probability in the heavy-tailed claim amounts. Then apply the Bühlmann credibility to estimate net premiums.

Pacáková (2013) applied Bayesian credibility analysis to estimate parameters for several statistical distributions given prior distribution. Furthermore, estimate the credibility premium or credibility number of claims in insurance.

Happ et al., (2014), applied Bühlmann-Straub credibility to claim to reserve non-life chain-ladder. They used multivariate credibility of N correlated portfolios to estimate the conditional mean square error of the ultimate claims and compared the estimated results with the multivariate additive method.

Jindrová (2014), demonstrated a classical Bayesian approach to estimate the probability of realizing the risk of death and permanent disability due to an accident for different age groups of men and women within the Slovak insurance market.

Seinerová (2015) illustrated and applied of Bühlmann Straub model to estimate the credibility costs by combining individual and collective experience throughout the credibility factor as a confidence level. This model is used for health care insurance.

Jindrová and Seinerová (2015), applied the Bühlmann Straub model to measure the cost of healthcare insurance and the corresponding credibility factor and concluded the for large companies, the estimation of the healthcare cost is reliable while for small-sized companies it's meaningless without the referral to data from the whole market. In addition, Bayesian analysis is considered a useful technique for healthcare insurance.

Gao (2016) illustrated modelling claim reserving using Bayesian analysis, this study classified into two sessions. The first session introduced Bayesian methodology for claim reserving. The second session proposed a compound model as a probabilistic approach and the Bayesian expansion models by applying Monte Carlo simulation for claim reserving.

Jindrová and Kopeck (2017), considered Bühlmann and Bühlmann-Straub empirical credibility to estimate the credibility premiums and net premiums for catastrophic claim amounts and economic losses for different regions. They applied Bühlmann and Bühlmann-Straub credibility for short-term insurance considering two types of data i.e. past data from risk itself and collateral data from other sources deemed relevant.

Hendrych and Cipra (2017) demonstrated the dynamic linear system of simultaneous equations for the non-life insurance market in Czech; they used these equations to estimate the desired variables i.e. (outstanding claims, unearned premiums, other technical provision and loadings). This approach might motivate the development of internal models applicable in the Solvency II framework.

1.2 Data and Methodology

Credibility theory is an actuarial approach used to calculate short term insurance premiums. This approach estimates premiums for each risk based on two ingredients: past data from the risk itself and data collected from other sources.

The Czech insurance market data was taken from the Czech Insurance Association; Tab. 1 illustrates the number of claims, and Tab. 2 illustrates the number of extreme losses for each risk area in three main types.

Tab. 1 shows the number of claims for three main types: damages caused by the weight of snow, damage caused by the floods and damage caused by gales and hail storms in the Czech insurance market. This table shows that the most significant damage was in 2013, driven by floods. Tab. 2 indicates the number of individual extreme events in the monitored period according to the risk area.

In this paper, we consider the development of Bühlmann and Bühlmann-Straub credibility models as a joint distribution of the claim amounts and the number of observed extreme losses to predict the upcoming year's net credible premium.

Tab. 1: The Amount of Extreme Claims (in thousands CZK)

Damages	2006	2007	2008	2009	2010	2011	2012
Damages caused by weight of snow	2 564 492	20 603	2 403	309 790	1 212 759	271 774	148 399
Damages caused by floods	1 340 848	386 892	5 070	1 508 902	3 994 437	336 827	353 794
Damages caused by gales and hail storms	685 606	3 134 566	1 250 653	1 936 736	2 706 853	1 045 302	1 740 007
Sum	4 590 946	3 542 061	1 258 126	3 755 428	7 914 049	1 653 903	2 242 200

Damages	2013	2014	2015	2016	2017	2018
Damages caused by weight of snow	124 402	22 070	20 704	19 976	105 474	13 591
Damages caused by floods	7 457 780	1 013 006	68 245	355 609	170 619	186 373
Damages caused by gales and hail storms	1 733 727	931 355	1 181 358	1 535 267	2 511 817	1 056 526
Sum	9 315 909	1 966 431	1 270 307	1 910 852	2 787 910	1 256 490

Source: authors' elaboration from (CIA, 2019)

Tab. 2: Number of Extreme Losses

Damages	2006	2007	2008	2009	2010	2011	2012
Damages caused by weight of snow	0	0	0	0	1	0	0
Damages caused by floods	0	0	0	1	1	0	0
Damages caused by gales and hail storms	0	1	1	0	1	0	0
Sum	0	1	1	1	3	0	0

Damages	2013	2014	2015	2016	2017	2018
Damages caused by weight of snow	0	0	0	0	0	0
Damages caused by floods	1	0	0	0	0	0
Damages caused by gales and hail storms	2	0	0	0	0	1
Sum	3	0	0	0	0	1

Source: authors' elaboration, data from (CIA, 2019)

One of the most important concerns for the insurance industry is the tail behaviour of loss distributions, where extreme events demonstrate the tail of loss distribution. This article introduced an approach that modified Bühlmann and Bühlmann-Straub credibility models by taking into account the tail behaviour of claims in order to predict credible premiums.

This paper aims to improve Bühlmann and Bühlmann-Straub credibility models to estimate the net credible premium as a linear function of the prior claims and the number of extreme events.

All statistical methods and calculations will be performed using the Statgraphics Centurion that is not an open-source statistical program, and R Package, an open-source environment for mathematical and statistical computations. From the R Package the {actuar}package is used to apply Bühlmann and Bühlmann-Straub credibility.

2 Credibility Models

2.1 Bühlmann Credibility Model

The Bühlmann credibility model assumes that statistical distribution, random variables $\{X_1, X_2 \ldots X_N, X_{N+1} \ldots\}$ are independently and identically distributed (*i.i.d*).

As mentioned above, the credibility premium consists of two ingredients, then

$$C = Z\,P_r + (1 - Z)\mu\ , \qquad (1)$$

where C represents the estimated pure premium, Pr referred to the estimation based on the prior data for each branch. Z is the credibility factor, which is a number between 0 and 1 that measure how much reliance the insurer is ready to face its own risk.

The Bühlmann credibility approach estimates the credibility premium from each risk; this model has been derived in Bühlmann (1976) and concluded the following results

$$E(m(\theta)|x) = Z\,\bar{x} + (1 - Z)E\big(m(\theta)\big)\ , \qquad (2)$$

and the credibility factor

$$\mathrm{Z} = \frac{n}{n + \frac{E(s^2(\theta))}{VAR(E(m(\theta)))}}\ , \qquad (3)$$

where $E(m(\theta))$ is the collective premium, $E(s^2(\theta))$ is within risk variance and $VAR(E\big(m(\theta)\big))$ between risk variance.

$$E(m(\theta)) = \bar{X} \qquad (4)$$

$$E(s^2(\theta)) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2, \qquad (5)$$

where

$m(\theta) = E[X_{ij}|\theta_i]$,

$s^2(\theta) = VAR[X_{ij}|\theta_i]$,

and

$$VAR(E(m(\theta))) = \frac{1}{N-1}\sum_{i=1}^{N}(\bar{X}_i - \bar{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2 . \qquad (6)$$

The derivations of the above quantities can be found in Bühlmann (1967).

The Bühlmann-Straub model doesn't assume that the random variables $\{X_1, X_2 \ldots X_N, X_{N+1} \ldots\}$ are independently and identically distributed (*i.i.d*) as an extent to Bühlmann credibility model.

2.2 Bühlmann-Straub Credibility Model

According to Bühlmann-Straub (1970), the estimation of the net credible premium for the *i*-th risk can be expressed as follows:

$$E(m(\theta)|x) = Z_i\,\bar{X}_i + (1 - Z_i)\bar{X} \qquad (7)$$

The credibility factor for each i risk Z_i is calculated from the formula

$$Z_i = \frac{P_i}{P_i + \frac{E(s^2(\theta))}{VAR(E(m(\theta)))}} \qquad (8)$$

where

$$E(s^2(\theta)) = \frac{1}{N(n-1)}\sum_{i=1}^{N}\sum_{j=1}^{n} P_{ij}(X_{ij} - \bar{X}_i)^2 \qquad (9)$$

$$VAR(E(m(\theta))) = P^* \left\{ \begin{matrix} \frac{1}{nN-1}\sum_{i=1}^{N}\sum_{j=1}^{n} P_{ij}(\bar{X}_{ij} - \bar{X})^2 \\ - \frac{1}{N(n-1)}\sum_{i=1}^{N}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2 \end{matrix} \right\} \qquad (10)$$

$$\bar{X}_i = \frac{1}{P_i}\sum_{j=1}^{n} P_{ij}\,X_{ij} = \frac{1}{P_i}\sum_{j=1}^{n} Y_{ij} \qquad (11)$$

$$\bar{X} = \frac{1}{P}\sum_{i=1}^{N} P_i\bar{X}_i \qquad (12)$$

$$P^* = \frac{1}{Nn-1}\sum_{i=1}^{N} P_i\left(1 - \frac{P_i}{P}\right) \qquad (13)$$

and P_{ij} is the number of extreme losses of the *i*-th risk in year *j*.

3 Results of the Research

Tab. 3 summarize the statistical characteristics of each insurance area in non-life insurance (extreme losses). This table displays the number of claims for the following risk areas: damages caused by wight of snow, damages caused by floods, and last risk area were damages caused by gales and hail storms. For damages caused by the weight of snow are mean in the amount of 372 034 000 CZK, the damages caused by floods are mean 1 321 420 000 CZK and damages caused by gales and hail storm is the mean 1 649 980 000 CZK (Benetti *et al*, 2019).

Tab. 3: Descriptive Analysis for Claim Amounts per Risk Area

Risk area	Mean (in thousands CZK)	Median (in thousands CZK)	St. Dev. (in thousands CZK)	Skewness	Kurtosis
Damages caused by the weight of snow	372 034 000	105 474 000	734 193 000	3.96215	5.39114
Damages caused by floods	1 321 420 000	355 609 000	2 132 260 000	3.57881	1.12532
Damages caused by gales and hail storm	1 649 980 000	1 535 270 000	748 197 000	4.38144	-0.25874

Source: Authors' calculations based on the result from Statgraphics Centurion.

To estimate net insurance premiums per risk area in 2019, it was necessary to estimate the parameters for each risk area (μ and σ). Furthermore, from equations (8) and (10), the average insured extreme events ($\overline{X_\iota}$), where $P_i = \sum_{j=1}^{n} P_{ij}$, $Y_i = \sum_{j=1}^{n} Y_{ij}$, results see in Tab. 4 (Benetti *et al*, 2019).

Tab. 4: Computed Characteristics of Branches

Branch	Total number of extremes (P_i)	Total amount of Claims (Y_i) (in thousands CZK)	$\overline{X_\iota}$ (in thousands CZK)
Damages caused by the weight of snow	1	4 834 034	4 834 034
Damages caused by floods	3	17 178 402	5 726 134
Damages caused by gales and hail storms	6	21 449 773	3 574 962

Source: Authors' calculations based on the result from R Package

Results for the calculated credibility factor Z and the estimation of net insurance premiums per each risk area in 2019 see in Tab. 5 (Benetti *et al*, 2019).

Tab. 5: Credibility Factors and Estimates of Net Insurance Premiums per risk area in 2019 (in thousands CZK)

Risk area	Credibility factor (Z_i)	Net Insurance Premium (in thousands CZK)
Damages caused by the weight of snow	0.7940475	1 486 558
Damages caused by floods	0.9204231	4 178 870
Damages caused by gales and hail storm	0.9585628	1 961 127

Source: Authors' calculations based on result from R Package.

Tab. 5 provides the net premiums for each risk area of insurance, showing how much money each area will need to cover extreme events in 2019. The estimates for net insurance premiums for damages were calculated for the Czech insurance market are 1 486 558 000 CZK for damages caused by the weight of the snow, 4 178 870 000 CZK for damage caused by floods, and finally, 1 961 127 000 CZK for damages caused by gales and hail storm.

3 Discussion

The future development of net premium can be modelled using various statistical modelling tools. Net insurance premiums were estimated using the Bühlmann-Straub Credibility Model.

The estimates for net insurance premiums for damages were calculated for the Czech insurance market are 1 486 558 000 CZK for damages caused by the weight of the snow, 4 178 870 000 CZK for damage caused by floods, and finally, 1 961 127 000 CZK for damages caused by gales and hail storm.

Data are now available on the Czech insurance market as to whether this modified credibility model is suitable for estimating net insurance premium. According to the results of the Czech Insurance Association, the results for 2019 were as follows: damages caused by the weight of snow was for 192 305 000 CZK, damages caused by floods was in the amount of 288 816 000 and damages caused by gales and hail storm was in the amount of 2 292 976 000 CZK.

The estimated values correspond to the actual values: by damages caused by the weight of snow only at 12.94%, by damages caused by floods even at only 6.91% and by damages caused by gales and hail storm 116.9%. Therefore, the most accurate estimate of the model is only in one of the three risk areas examined, namely damages caused by gales and hail storm 116.9%.

It is evident that if the insurers used this net premium estimation model, they would create sufficient reserves to cover the risks. Of course, by law, they must make compulsory reserves, which are calculated based on well-defined procedures, whereby insurance companies have to count on maximum damage. This model does not calculate as much damage as possible, but the expected net premium is estimated to cover the costs of the insurance company with the appropriate type of risk. The results show that the estimation of values compared to real values was very inaccurate in two risk areas - although it would allow insurers to create sufficient reserves, but it can be said that they are exaggerated. Here it would be appropriate to consider whether, even when applying this model, the insurer would still be a "good financial manager", as required by law. However, it can be clearly stated that this model can serve insurance companies as a complementary model for estimating net premiums.

Conclusion

Bühlmann and Bühlmann-Straub credibility approaches represent a recent development of the Bayesian credibility theory; these models apply the greatest accuracy theory. In this study we improved Bühlmann and Bühlmann-Straub credibility models as a joint distribution in order to estimate the net credible premium using incurred claims and extreme losses. However, we adopt Bühlmann-Straub credibility because it doesn’t assume that risks are independently and identically distributed (i.i.d). This research concentrates on three risk areas in the Czech non-life insurance market during the period 2006 to 2018. Furthermore, these branches include extreme events to predict the credibility net premium for the upcoming year 2019 for each risk area in the Czech insurance market, as shown in Tab. 4. Moreover, this process shows how much money each branch will need to cover extreme events to manage risks.

Future research may consider fluctuation in claim amounts resulting from extreme losses or lapses in insurance policies, based on the economic changes in the Czech insurance market.

Acknowledgement

This paper was created with the support of specific research by EF TUL 2021-2022 (the project of Internal Grant Competition “Use of "Credibility Theory").

References

Bayes, T. (1763). An Essay towards solving a Problem in the Doctrine of Chances. Phil. Trans, 1763, 53, 370–418.

Benetti, K.; Elsayed, M.; Soliman, A. & Khalil, D. (2019). Credibility Modelling for Extreme Losses of Natural Hazards in the Czech Republic: An Actuarial Approach. *In Liberec Economic Forum. Proceedings of the 14th International Conference*, Liberec: Technical University of Liberec, 2019, pp. 81–92. ISBN 978-80-7494-482-6. WOS: 000540987200007.

Bühlmann, H. & Straub, E. (1970). Glaubwürdigkeit für Schadensätze. *Bulletin of the Association of Swiss Actuaries*, 1970, 70(1), 111-133.

Bühlmann, H. (1967). Experience rating and credibility. *ASTIN Bull*, 1967, 4, 199-207.

CIA. (2019). *Škody z pojištění majetku* [online]. 2019 [cit. 2019-04-05]. Available at: http://www.cap.cz/statisticke-udaje/skody-z-pojisteni-majetku

Gao, G. (2016). *Three Essays on Bayesian Claims Reserving Methods in General Insurance*. The Australian National University, PhD Thesis, 2016.

Happ, S., Maier, R. & Merz, M. (2014). Multivariate Bühlmann-Straub Credibility Model Applied to Claims Reserving for Correlated Run-off Triangle. *Casualty Actuarial Society*, 2014, 8(1): 23-42.

Hendrych, R. & Cipra, T. (2017). Econometric model of non-life technical provisions: the Czech insurance market case study. *European Actuarial Journal*, 2017, 7, 257-276. WOS:000406907700010.

Jindrová, P. & Kopeck, L. (2017). Empirical Bayes Credibility Models for Economic Catastrophic Losses by Regions. DOI: 10.1051/70901006itmconf/201, 2017.

Jindrová, P. & Seinerová, K. (2015). Bayesian estimates of health care costs in Slovakian insurance companies. *International Journal of Economics and Statistics*, 2015, 3: 174-177.

Jindrová, P. (2014). Credibility Risk Models in Accident Insurance. *Managing and Modelling of Financial Risks: 7th International Scientific Conference, PTS I-III* Ostrava, 307-316. WOS:000350605800037.

Linda, B. & Kubanová, J. (2012). Credibility Premium Calculation in Motor Third-Party Liability Insurance. *Proceedings of the 14th Wseas International Conference on Mathematical and Computational Methods in Science and Engineering*, Sliema, Malta. September 7-9, 2012, 259-263.

Loisel, S. & Trufin, J. (2013). Ultimate Ruin Probability in Discrete Time with Bühlmann Credibility Premium Adjustments. *Bulletin Francais D'actuariat,* 2013, 13(25), 73-102.

Pacáková, V. P. (2013). Credibility models for permanently updated estimates in insurance. *International Journal of Mathematical Models and Methods in Applied Sciences*, 2013, 7(3), 333-340.

Seinerová, K. (2015). Segment-specific Bayesian analysis of health insurance costs. *10th International Scientific Conference Financial Management of Firms and Financial Institutions.* Ostrava, VŠB-TU of Ostrava, Faculty of Economics, Department of Finance, 2015, 1115-1120. WOS:000376799500136.

Contact

Karina Benetti

Technical University of Liberec, Faculty of Economics

Department of Economic Statistics

Studentska 1402/2

461 17 Liberec

Czech Republic

karina.benetti@tul.cz