POSSIBILISTIC FUZZY FUNCTIONS FOR FORECASTING STOCK EXCHANGE INDICES

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Abstract

Fuzzy functions (FFs) were introduced by Turksen as non-rule-based inference systems. The idea behind FFs is to cluster the observations by using Fuzzy c-means (FCM) algorithm. Thus, an object belongs to each cluster with some membership grade. Turksen claims that adding the membership grades into input matrix for each cluster as a new variable and combining the outcomes would improve the performance of the maximum likelihood estimator. Moreover, he also shows that adding different functions of membership grades into inputs improves the model performance even better. However, adding different functions of membership grades causes the multicollinearity problem. To overcome the problem, elastic-net estimators are adapted in fuzzy functions by Tak and İnan [2]. They used FCM to cluster the inputs and obtain the membership grades. However, FCM calculates the centers of clusters by using all observations. Thus, FCM is very sensitive when there are outliers in the datasets. To deal with misspecification of the cluster centers, Possibilistic FCM (PFCM) is employed in fuzzy functions with elastic-net estimators. The proposed method is evaluated on three real world datasets. The results verified that the proposed method outperformed the other methods that are used in the study.

Key words: T1FF, Elastic-net, Possibilistic FCM, Forecasting, Fuzzy Logic

JEL Code: C45, C53

Introduction

Forecasting plays a pivotal role in decision-making by systematically predicting future events or trends using historical data. This methodical approach utilizes mathematical models, statistical tools, and diverse techniques to anticipate future occurrences. Whether used in business, economics, meteorology, or other disciplines, forecasting offers a structured framework for planning and decision-making amid uncertainties. By analyzing historical data patterns and trends, forecasting facilitates informed predictions.

Forecasting techniques are categorized into two main groups: statistical and alternative methods. Statistical methods, which excel when the data meets particular assumptions, are based on simple models and deterministic principles. However, these methods often fail to produce satisfactory results when dealing with complex, dynamic, and unpredictable real-world situations.

Recent literature on forecasting has explored alternative methods such as artificial intelligence and fuzzy logic-based approaches, which depart from traditional techniques to improve prediction accuracy and flexibility. Artificial intelligence, especially through sophisticated machine learning techniques like artificial neural networks (McCulloch and Pitts, 1943), excels at identifying complex patterns and capturing nonlinear relationships in data. This capability allows forecasters to manage intricate and dynamic information, providing a deeper understanding of trends and behaviors. Fuzzy logic, meanwhile, is useful for handling imprecise and uncertain data by representing varying degrees of membership and linguistic terms, making it particularly effective where traditional forecasting models struggle with ambiguity.

Fuzzy inference systems (FIS), introduced by (Zadeh, 1973) are powerful artificial intelligence tools designed to model and reason with uncertainty and imprecise information. Their core functionality depends on expert-defined fuzzy rules that capture relationships between input and output variables. These rules, typically in "IF-THEN" format, use fuzzy conditions on input variables to determine the desired output. Therefore, the performance of an FIS is closely tied to the quality and accuracy of the expert knowledge in its rule base. This reliance limits traditional FIS in their ability to learn and adapt automatically to changing environments, which is a significant drawback in dynamic systems. To overcome this limitation and improve FIS learning capabilities, (Turksen, 2008) proposed type-1 fuzzy function-based approaches that enable automatic rule learning.

Type-1 Fuzzy Functions (T1FFs) are based on a regression model that uses membership values and their transformations as predictors. However, this often leads to multicollinearity, which violates the assumptions for Ordinary Least Squares (OLS) regression. In regression modeling, multicollinearity can cause issues such as high variance in parameter estimates and overfitting. While Ridge regression can address the high variance, it is insufficient for mitigating overfitting. Elastic Net stands out from Lasso and Ridge by effectively managing both problems. (Bas et al., 2019-2020), (Kizilaslan et al., 2020) and

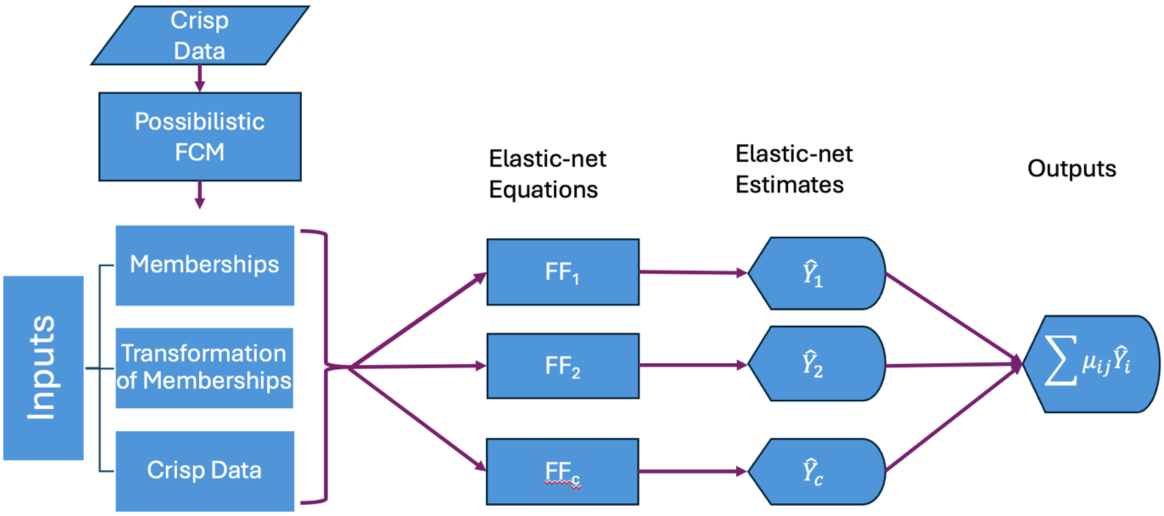
(Tak and Inan, 2022) addressed the multicollinearity problem in their studies by using Ridge Regression. Although ridge regression can resolve multicollinearity, it does not address the issue of overfitting.

Besides, T1FFs utilize Fuzzy C-Means (FCM) to determine the membership degrees included in the input matrix. Clustering is crucial for identifying accurate patterns, leading to extensive research on clustering algorithms. Many studies aim to identify the right patterns. FCM, introduced by (Dunn, 1973) and further developed by (Bezdek, 1984) is among the most used fuzzy clustering techniques. However, FCM has limitations in determining cluster centers and membership degrees. To address these limitations, several studies have been conducted Krishnapuram and Keller, 1993) and (Pal, Pal, Keller and Bezdek, 2005). (Krishnapuram and Keller, 1997) proposed Possibilistic C-Means (PCM) to tackle issues like overlapping clusters, sensitivity to initialization, and noise. Some studies in the literature that uses different clustering algorithms are proposed by (Tak, 2020a and 2020b), (Bas and Egiroglu, 2024) and (Egrioglu and Bas, 2023) To address these challenges in time series prediction, Type-1 Possibilistic Fuzzy Functions with Elastic-Net Regularization (E-T1PFF) are proposed in the study.

1 Proposed Method

A key task for clustering methods is to differentiate distinct objects into separate groups heterogeneously while grouping similar objects together homogeneously. Accurate determination of cluster centers is essential in this process. Fuzzy C-Means (FCM) considers each observation when determining cluster centers. Consequently, if a dataset contains outliers or noise, the cluster centers can be significantly affected, potentially resulting in overlapping clusters. Besides, T1FFs incorporate membership grades and their nonlinear transformations into the input matrix, using the least squares method to estimate model coefficients. However, including these related variables in the input matrix violates the least squares assumption that there should be no linear relationship between explanatory variables, leading to overfitting problems. Aforementioned limitations brought us to come with the proposed method, Possibilistic Fuzzy Functions with Elastic-net regularization for forecasting stock exchange indices. The detailed steps and the flowchart of the proposed method are given below.

Fig. 1: Architecture of Possibilistic Fuzzy Functions with Elastic-net Regularization



"Source: Own research"

Algorithm 1. The Proposed Method

Step 1. The inputs are the lagged values of the given time series.

$$X = [Y_{t-1} \ \ Y_{t-2} \ \ldots \ Y_{t-p}] \quad (1)$$

Step 2. The hyperparameters are initialized.

c: number of clusters

p: the lag numbers

λ: L1 penalization coefficient

α: L2 penalization coefficient

m: fuzziness index

The hyperparameters c,p,λ,α are determined by grid search, and m is set to 2.

Step 3. Possibilistic FCM is used to cluster inputs and determine the membership grades (μ) of the observations and cluster centers (v).

Step 4. Membership grades (μ) and their transformations of the observations are included in the input matrix for each cluster.

$$X_i = \left[\mu_i \, , e^{\mu_i}, \log(\mu_i) \, , Y_{t-1} \ \ Y_{t-2} \ \ldots \ Y_{t-p}\right], i = 1,2, \ldots, c \quad (2)$$

Step 5. Elastic net regression is trained by using the input matrix for each cluster.

Step 6. The outputs are the weighted combination of the forecasts. The weights/membership grades are obtained by using cluster centers and the new observations.

2 Applications

3 time series datasets from Stock Exchange of Istanbul are used. The elements of these time series datasets are observed for the first half of the years of 2009, 2010 and 2011. The time series data is split into three sets: training, validation, and test sets. The training set is utilized to train the model, the validation set is employed to select the methods' hyperparameters, and the test set is used to evaluate the methods' performance. The lengths of test sets are set to 7 and 15. The optimum value of c and p are searched between 2-10. Besides, optimum values of α and λ are searched between 0 - 1 with increment of 0.1 and 0 and 100 with increment of 0.1 , respectively. Table 1 gives the optimum values of hyper parameters after the search.

Tab. 1: Hyperparameter detections

Datasets	length of test	p	c	α	λ
2009	7	3	2	0	1
2009	15	2	2	0	12.59
2010	7	9	7	0	100
2010	15	9	9	0	100
2011	7	2	3	0.6	10
2010	15	5	6	0	100

"Source: Own research"

Inspecting Table 2 and Table 3, the proposed method outperforms the other methods in both MAPE and RMSE values.

$$RMSE = \sqrt{\frac{1}{ntest}\sum_{t=1}^{ntest}(y_t - \hat{y}_t)^2} \quad (3)$$

Table 2: MAPE values of the Methods

Datasets	length of test	ARIMA	ES	MLP-ANN	T1FFs	MAPE
2009	7	0.0087	0.0087	0.0083	0.0101	**0.0081**
2009	15	0.012	0.012	**0.0114**	0.0122	0.0115
2010	7	0.0183	0.0185	0.0143	0.0179	**0.0134**
2010	15	0.022	0.022	0.022	0.0264	**0.0218**
2011	7	0.0144	0.0144	0.0128	0.0153	**0.0120**
2010	5	0.015	0.015	0.0146	0.0156	**0.0144**

"Source: Own research"

Table 3: RMSE values of the methods

Datasets	length of test	ARIMA	ES	MLP-ANN	T1FFs	RMSE
2009	7	345	345	325	446	**320**
2009	15	540	540	525	534	**511**
2010	7	1221	1208	**1077**	1180	1089
2010	15	1612	1612	1603	1852	**1497**
2011	7	1058	1057	**920**	1083	1009
2010	5	1130	1130	1096	1146	**1049**

"Source: Own research"

Conclusion

In this study, Possibilistic type-1 fuzzy functions with elastic-net as a new fuzzy inference system for forecasting is proposed. A training algorithm based on elastic-net regression to eliminate the reductant variables. The performance of the proposed method is compared with some popular alternative and classical time series methods for stock exchange time series datasets of Istanbul. It is observed that the proposed method is able outperforms the other methods. In future studies, it is planned to evaluate the performance of the proposed method with more time series datasets and increase the search space of the hyperparameters.

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References

Bas, E., Egrioglu, E., Yolcu, U., & Grosan, C. (2019). Type 1 fuzzy function approach based on ridge regression for forecasting. Granular Computing, 4, 629-637.

Bas, E., & Egrioglu, E. (2024). Robust Picture Fuzzy Regression Functions Approach Based on M-Estimators for the Forecasting Problem. Computational Economics, 1-36.

Bas, E., Yolcu, U., & Egrioglu, E. (2020). Picture fuzzy regression functions approach for financial time series based on ridge regression and genetic algorithm. Journal of Computational and Applied Mathematics, 370, 112656.

Bezdek, J. C., Ehrlich, R., & Full, W. (1984). FCM: The fuzzy c-means clustering algorithm. Computers & geosciences, 10(2-3), 191-203.

Dunn, J. C. (1973). A fuzzy relative of the ISODATA process and its use in detecting compact well-separated clusters.

Egrioglu, E., & Bas, E. (2023). Robust intuitionistic fuzzy regression functions approaches. Information Sciences, 638, 118992.

Kizilaslan, B., Egrioglu, E., & Evren, A. A. (2020). Intuitionistic fuzzy ridge regression functions. Communications in Statistics-Simulation and Computation, 49(3), 699-708.

Krishnapuram, R., & Keller, J. M. (1993). A possibilistic approach to clustering. IEEE transactions on fuzzy systems, 1(2), 98-110.

McCulloch, W. S., & Pitts, W. (1943). A logical calculus of the ideas immanent in nervous activity. The Bulletin of Mathematical Biophysics, 5 (4), 115–133. doi: 10. 1007/BF02478259

Pal, N. R., Pal, K., Keller, J. M., & Bezdek, J. C. (2005). A possibilistic fuzzy c-means clustering algorithm. IEEE transactions on fuzzy systems, 13(4), 517-530.

Tak, N. (2020). Type-1 recurrent intuitionistic fuzzy functions for forecasting. Expert systems with applications, 140, 112913.

Tak, N. (2020). Type-1 possibilistic fuzzy forecasting functions. Journal of Computational and Applied Mathematics, 370, 112653.

Tak, N., & İnan, D. (2022). Type-1 fuzzy forecasting functions with elastic net regularization. Expert Systems with Applications, 199, 116916.

Türkşen, I. B. (2008). Fuzzy functions with LSE. Applied Soft Computing, 8(3), 1178-1188.

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