QUANTILES VS. TANTILES: A COMPARISON OF VARIABILITY IN WAGE DISTRIBUTIONS

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Abstract

This paper examines wage inequality and gender differences in wage distributions using both traditional quantile measures and the newly introduced tantile approach. While quantiles describe the distribution of employees according to wage levels, tantiles are based on cumulative shares of total wage income and therefore capture income concentration more effectively, particularly in the upper tail of the distribution. The main objective of the study is to compare the development of quantile- and tantile-based measures of variability in the Czech Republic during the period 2009–2024 for the total population as well as separately for men and women. The analysis employs selected quantiles and tantiles together with measures of absolute and relative variability derived from the 1%, 10%, 25%, 75%, 90%, and 99% levels. The results indicate that tantile-based measures systematically exceed the corresponding quantile measures, especially in higher income groups, revealing stronger concentration effects than those captured by conventional quantiles. Wage variability increased in absolute terms over the observed period, with higher inequality among men, while relative variability showed a slight decline. The findings demonstrate that tantiles provide a complementary and more informative perspective on wage inequality and gender wage disparities.

Key words: Quantiles, tantiles, wage distribution, income inequality, variability measures

JEL Code: C46, C43, D31

Introduction

Research on wage inequality and gender wage gaps increasingly uses quantile-based methods to analyze wage distributions. Koenker and Bassett (1978) introduced regression quantiles, while Firpo, Fortin, and Lemieux (2009) developed unconditional quantile regressions. Recent studies extended these approaches to gender wage differences (Anastasiade-Guinand, Matei, and Tillé, 2023), selection bias (Batbayar et al., 2025), and weighted-average quantile

regressions (Chetverikov, Liu, and Tsyvinski, 2025). Other research addressed robustness and inference issues in inequality measurement, including the sensitivity of inequality indices to extreme values (Cowell and Flachaire, 2007), distribution-free estimators for quantile ratios (Prendergast and Staudte, 2017), estimators for heavy-tailed distributions (Jędrzejczak, Pekasiewicz, and Zieliński, 2023; Kebe et al., 2023), and inference for quintile share ratios (Langel and Tillé, 2011). Quantile methods have also been applied to minimum wage effects (Bossler and Schank, 2023), historical wage data (Clarke, Llorca Jaña, and Pailañir, 2023), global income inequality (Lakner and Milanovic, 2013), climate policy effects (Rüb, 2024), and grouped-income estimation (Spasova, 2024). Despite this extensive literature, research mainly focuses on quantile-based measures. This study introduces tantiles, based on cumulative income shares, as a complementary tool capturing income concentration more effectively than traditional quantiles. The paper compares tantile- and quantile-based measures of absolute and relative variability using wage distributions in the Czech Republic for 2009–2024, both for the total population and separately for men and women.

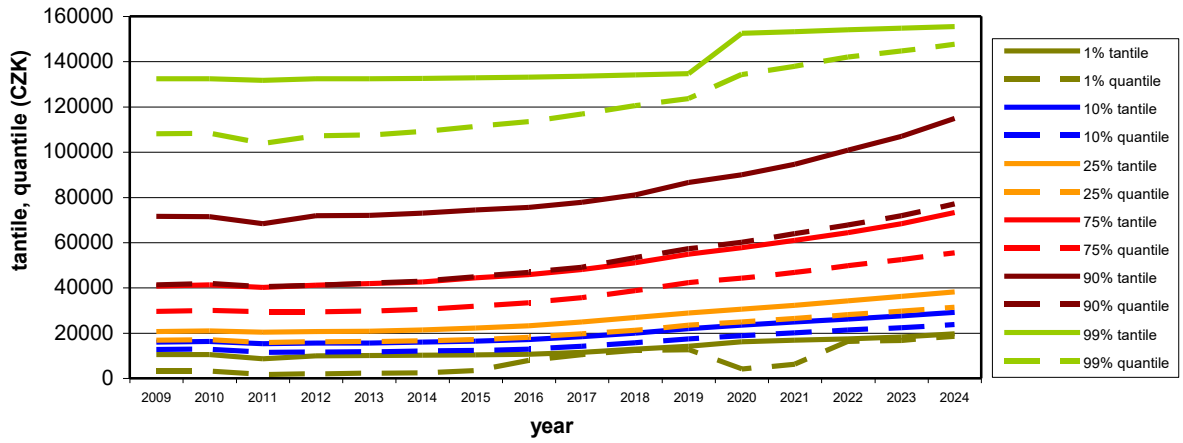
1 Theory and Methods

P% quantile $\tilde{x}_p, P \in (0;100)$, is the value of the variable *x*, such that *P*% of the values of this variable are less than or equal to $\tilde{x}_p$ and the remaining (100 – *P*)% of the values of this variable are greater than or equal to the value $\tilde{x}_p$. *P*% tantile $\check{x}_p, P \in (0;100)$, is the value of the variable *x*, such that the sum of values less than or equal to $\check{x}_p$ represents *P*% of the total sum of all values of this variable, and the sum of values greater than or equal to $\check{x}_p$ represents the remaining (100 – *P*)% of the total sum of all values of this variable.

The following measures of absolute variability: difference between 99% tantile and 1% tantile, difference between 90% tantile and 10% tantile, difference between 75% tantile and 25% tantile, difference between 99% quantile and 1% quantile, difference between 90% quantile and 10% quantile and difference between 75% quantile a 25% quantile; and measures of relative variability: difference between 99% tantile and 1% tantile divided by 99% tantile, difference between 90% tantile and 10% tantile divided by 90% tantile, difference between 75% tantile and 25% tantile divided by 75% tantile, difference between 99% quantile and 1% quantile divided by 99% quantile, difference between 90% quantile and 10% quantile divided by 90% quantile and difference between 75% quantile a 25% quantile divided by 75% quantile. After multiplying by one hundred, these characteristics are expressed in percentages.

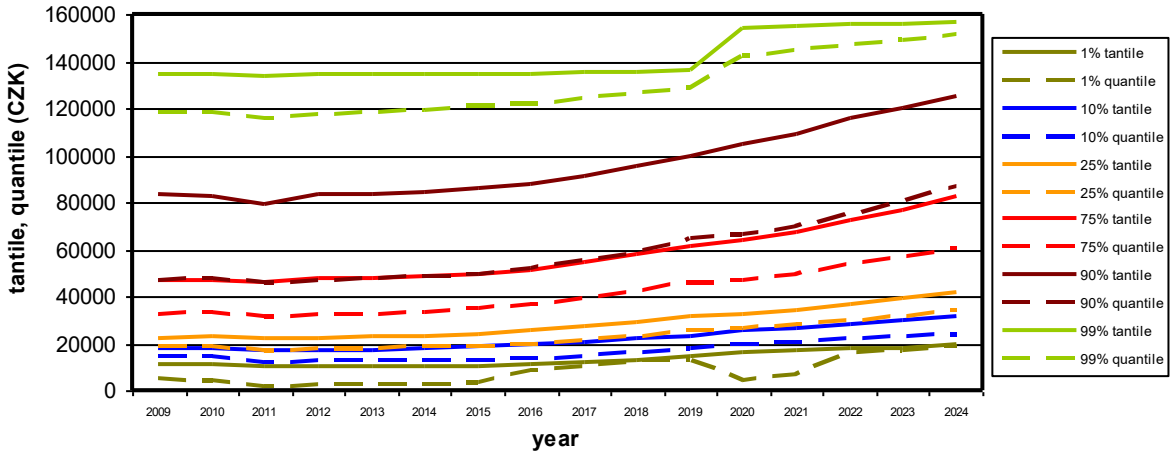
2 Results and Conclusion

Fig. 1: Development of tantiles and quantiles: Total



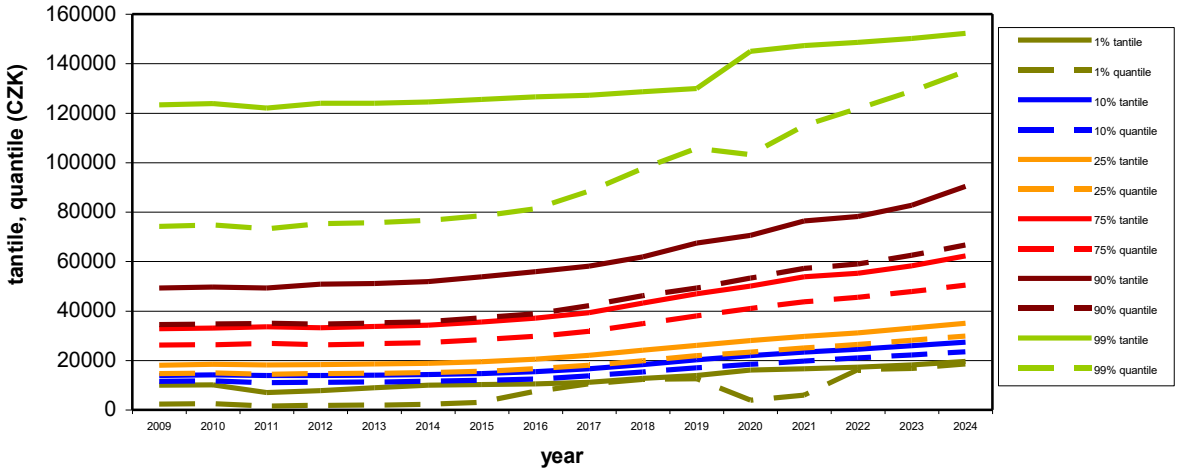
Source: Own research

Fig. 2: Development of tantiles and quantiles: Men



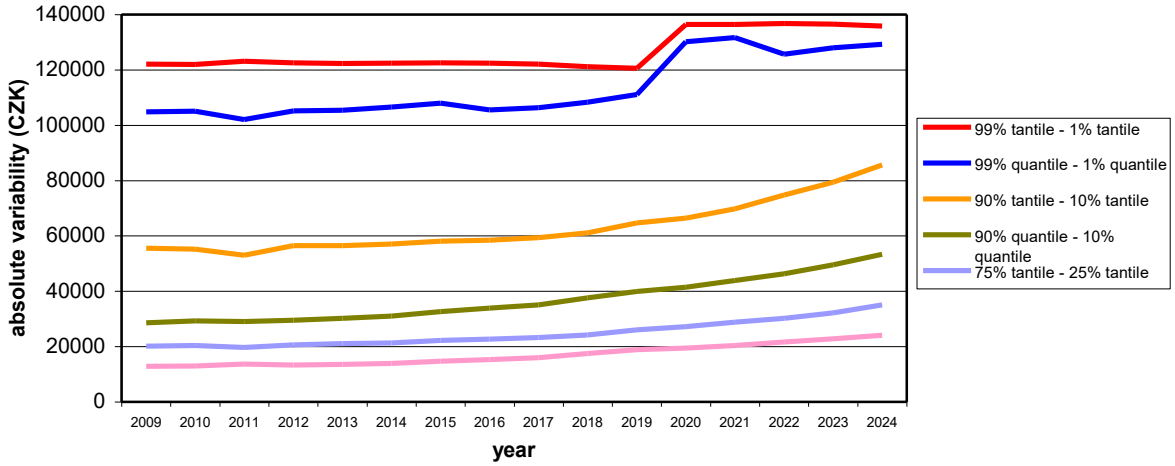
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Fig. 3: Development of tantiles and quantiles: Women



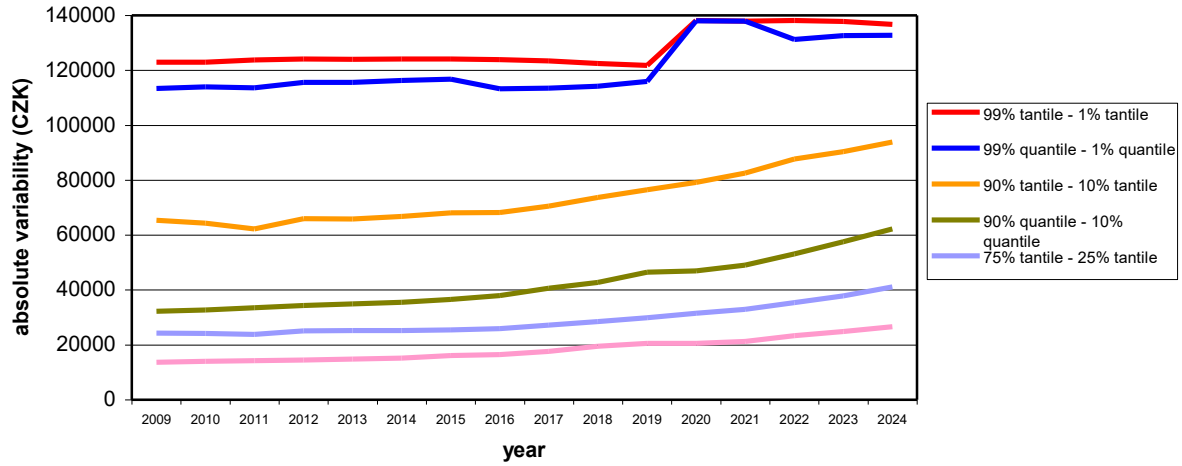
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Fig. 4: Development of absolute variability: Total



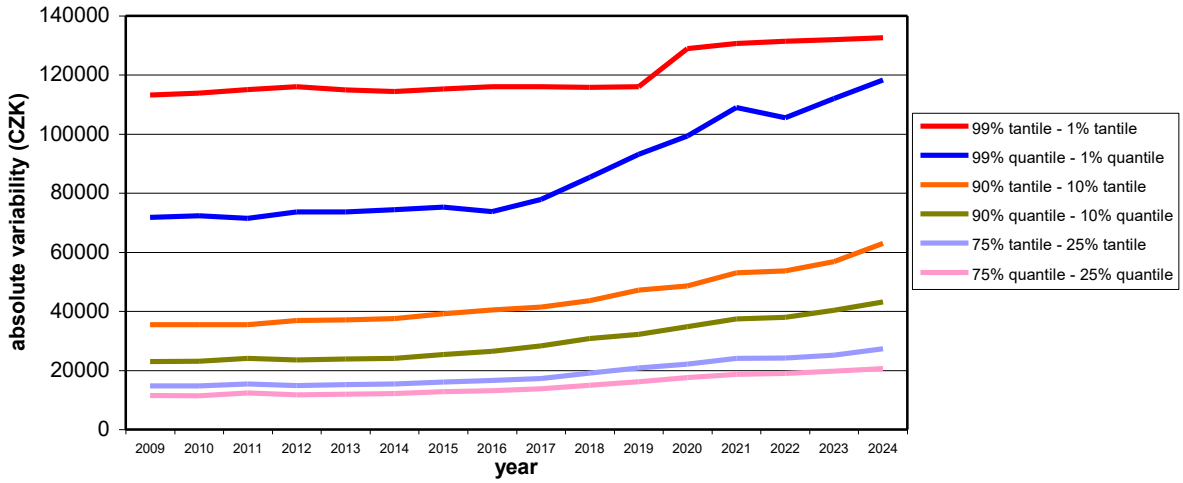
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Fig. 5: Development of absolute variability: Men



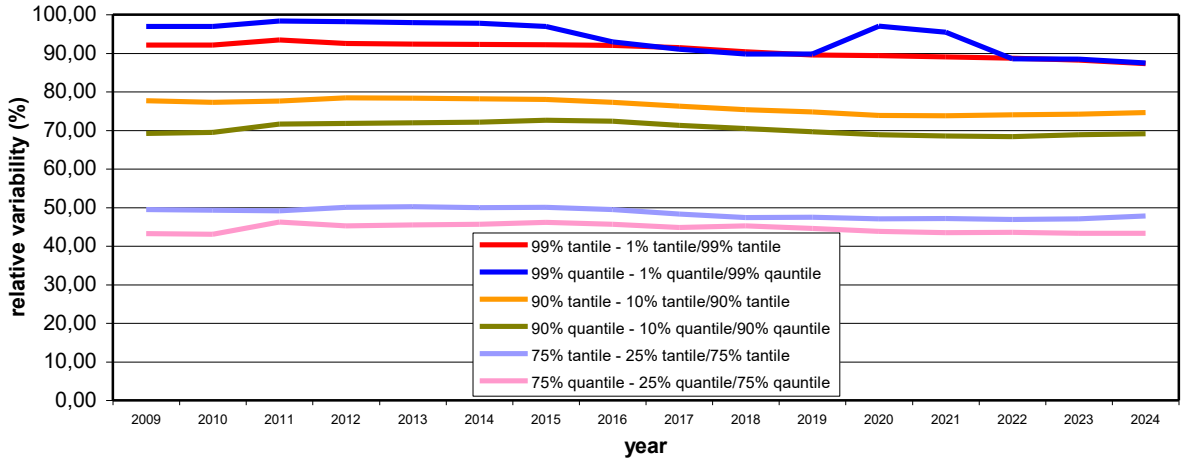
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Fig. 6: Development of absolute variability: Women



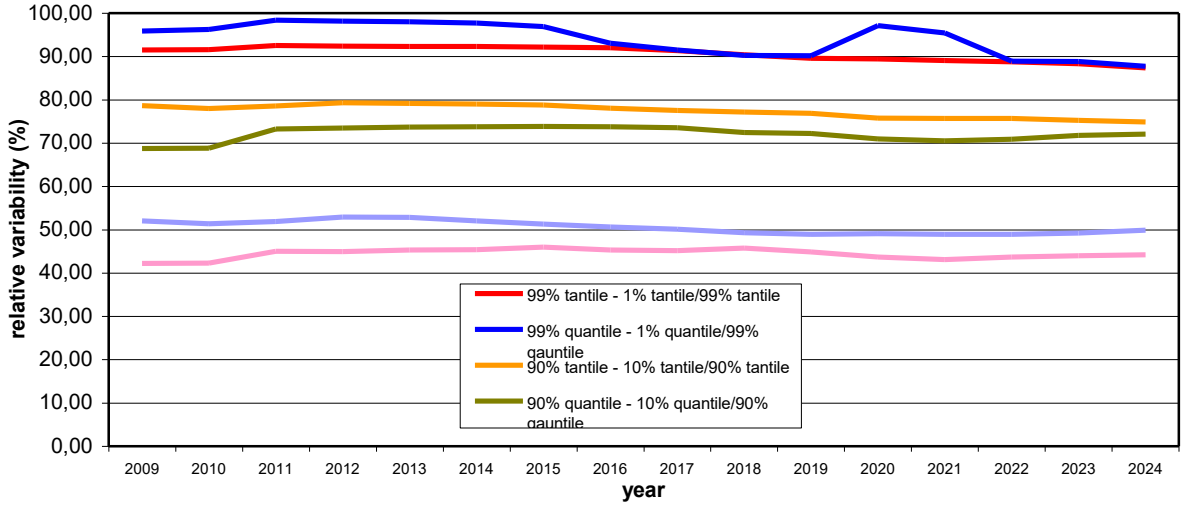
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Fig. 7: Development of relative variability: Total



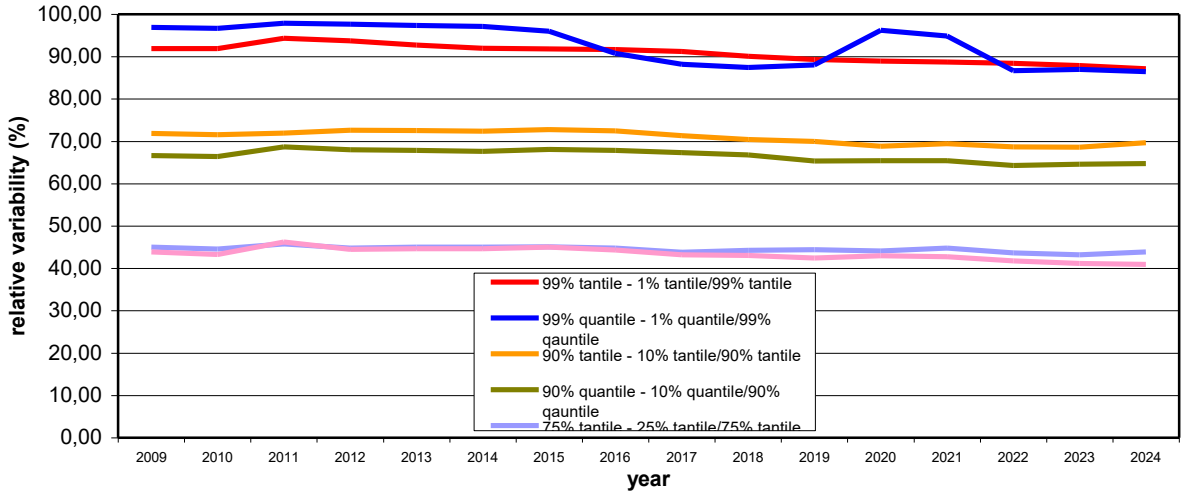
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Fig. 8: Development of relative variability: Men



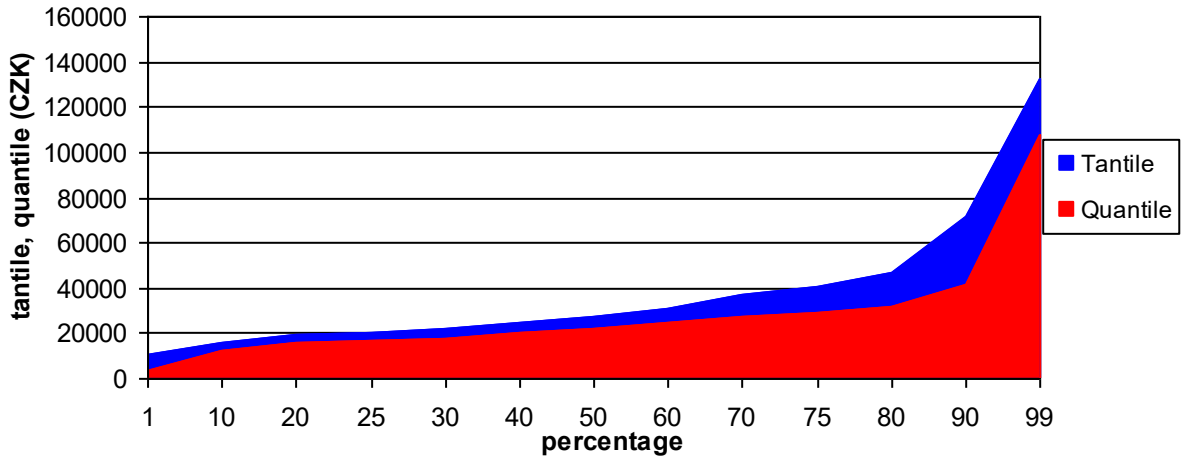
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Fig. 9: Development of relative variability: Women



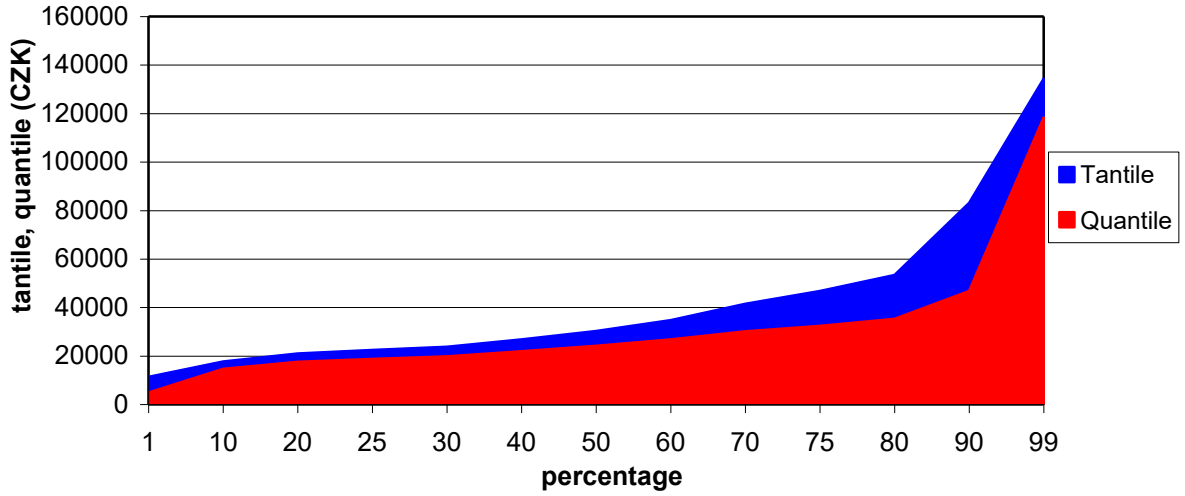
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Fig. 10: Tantile and quantile values in 2009: Total



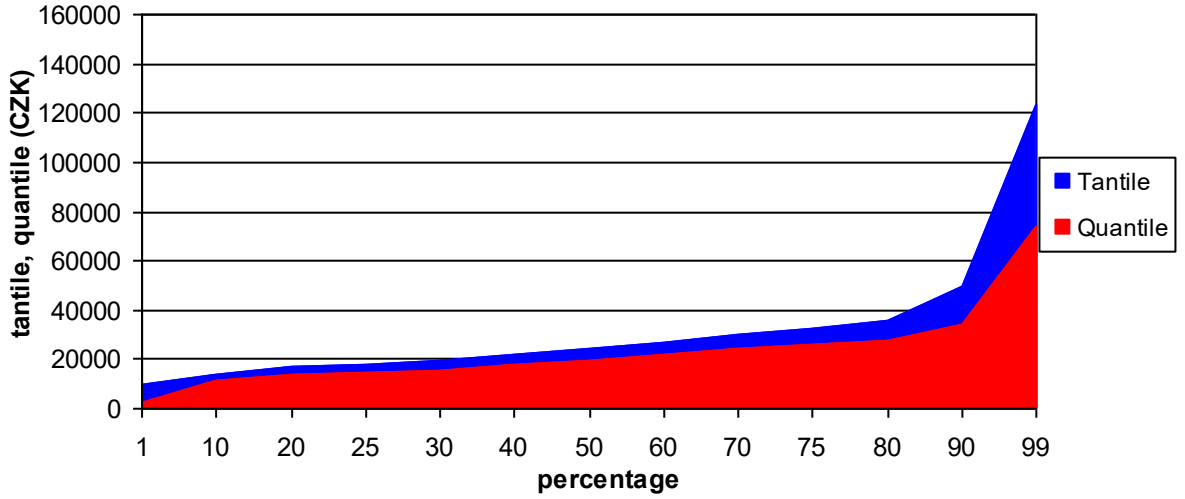
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Fig. 11: Tantile and quantile values in 2009: Men



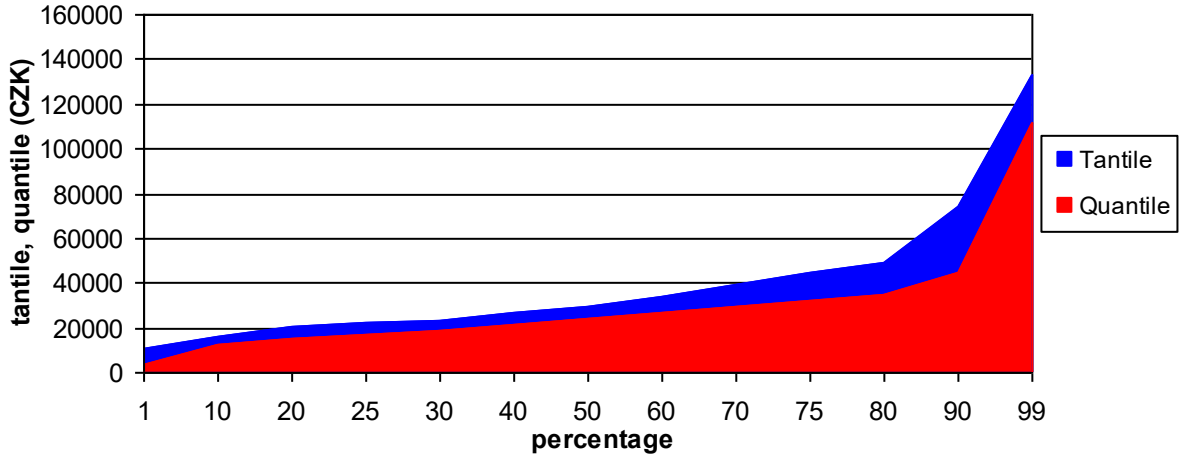
Source: Own research

Fig. 12: Tantile and quantile values in 2009: Women



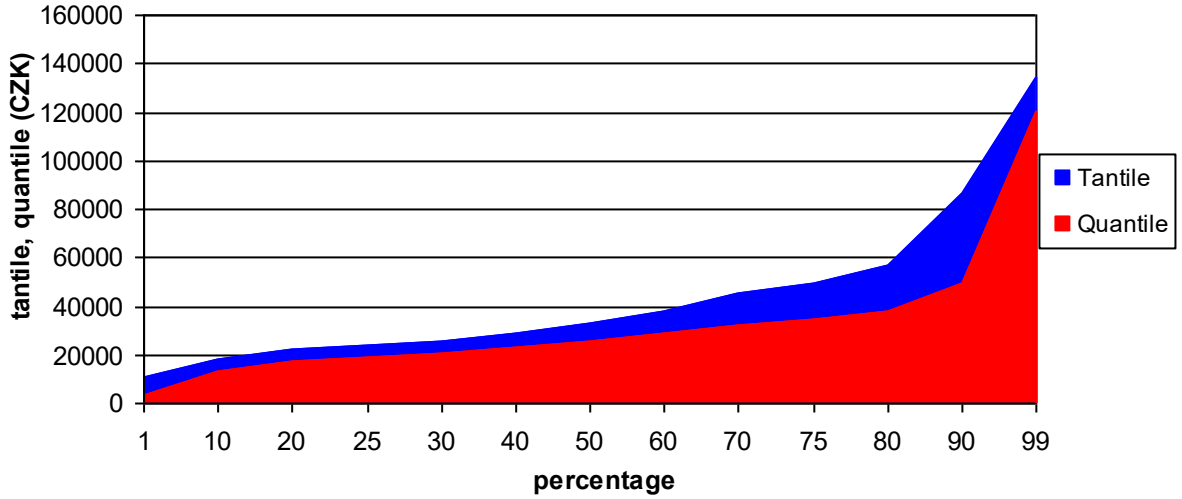
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Fig. 13: Tantile and quantile values in 2015: Total



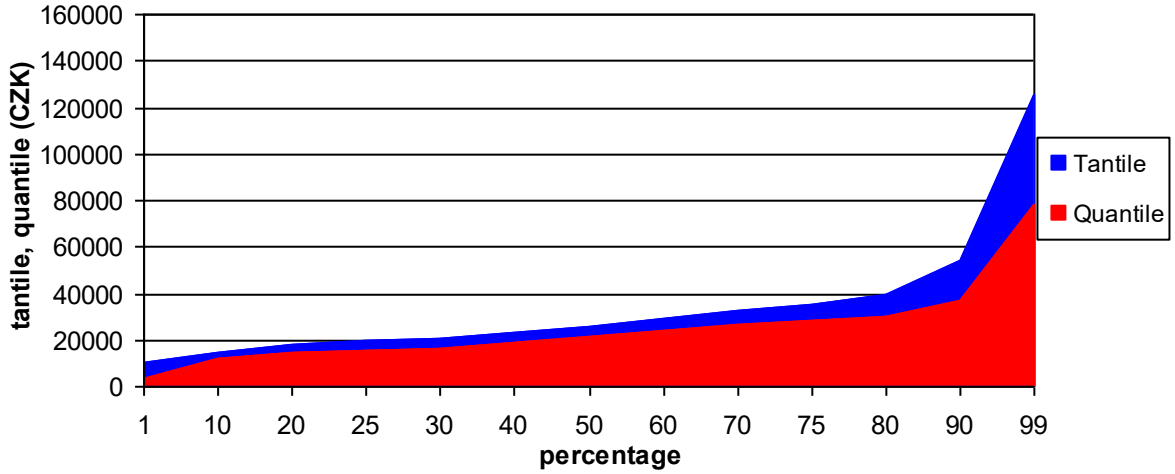
Source: Own research

Fig. 14: Tantile and quantile values in 2015: Men



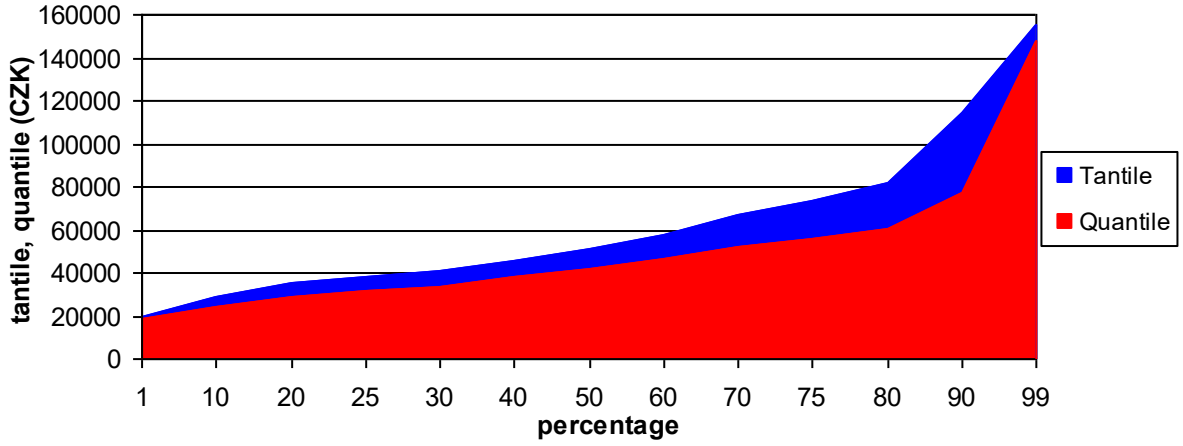
Source: Own research

Fig. 15: Tantile and quantile values in 2015: Women



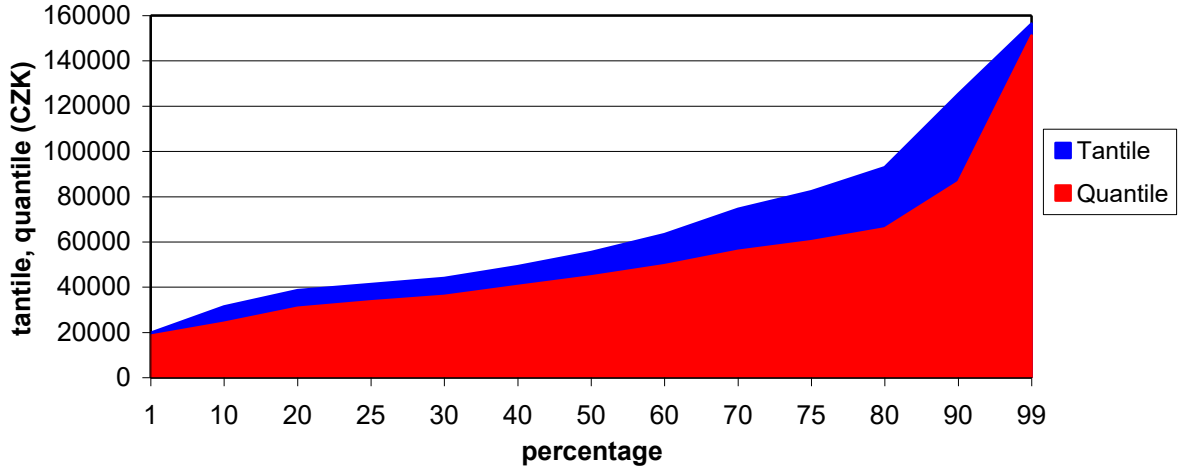
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Fig. 16: Tantile and quantile values in 2024: Total



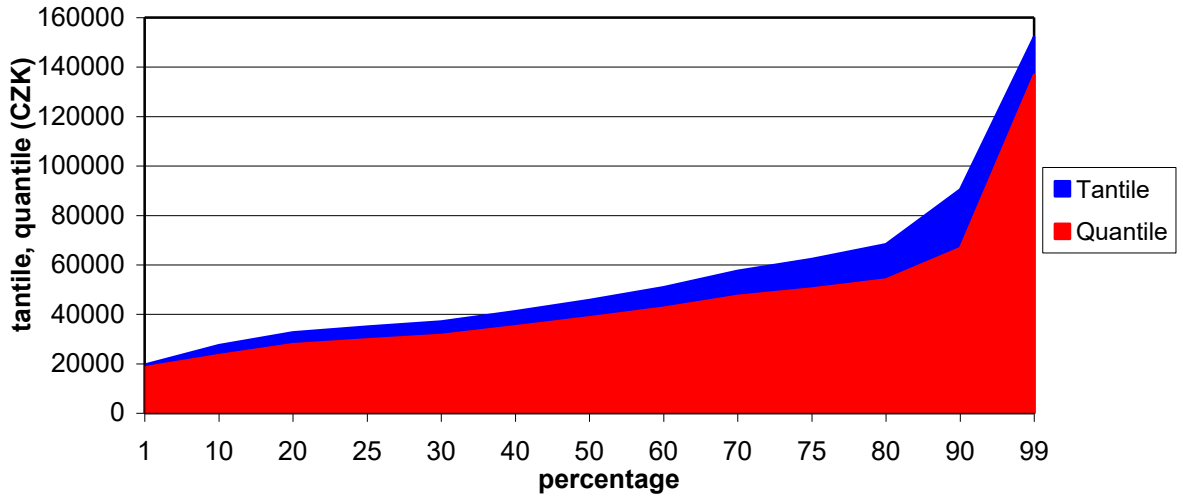
Source: Own research

Fig. 17: Tantile and quantile values in 2024: Men



Source: Own research

Fig. 18: Tantile and quantile values in 2024: Women



Source: Own research

The research results are presented in Figures 1–18. The measures of absolute variability also confirm growing wage differentiation. In the total sample, the difference between the 99% and 1% tantiles increased from CZK 122,042 in 2009 to CZK 135,798 in 2024, while the corresponding quantile difference rose from CZK 104,775 to CZK 129,211. Tantile-based measures of variability are consistently higher than the corresponding quantile measures. The highest absolute variability is consistently observed among men, particularly in indicators based on the 90% and 99% characteristics. Relative variability, in contrast, shows a slight downward trend. For the total sample, the ratio [(99% tantile – 1% tantile)/99% tantile] declined from 92.13% in 2009 to 87.34% in 2024. A similar decline is visible for the corresponding quantile indicators. This suggests that although absolute wage and salary differences increased, relative disparities compared with the highest wages became slightly smaller over time. Relative variability remains consistently higher among men than among women, confirming greater wage inequality within the male population.

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