MODERN TECHNOLOGIES IN MATHEMATICS EDUCATION

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Abstract

The development of artificial intelligence tools has created new opportunities for mathematics education at universities. In addition to generative artificial intelligence, the Python programming language and modern educational platforms that support active student engagement can also be effectively incorporated into the teaching process. This paper presents possibilities for integrating these tools into mathematics courses at the Prague University of Economics and Business. The paper discusses examples of the use of generative AI for explaining mathematical procedures, creating learning materials, and providing individualized feedback to students. It also demonstrates the use of Python for the visualization of mathematical concepts. Particular attention is devoted to the FeedbackFruits platform, which enables the implementation of interactive learning activities, continuous assessment of students' understanding, and support for collaborative learning. The aim of the paper is to provide an overview of practical applications of these tools in mathematics education and to inspire instructors to implement them in a meaningful and effective way. The paper includes examples of activities suitable for university-level mathematics courses.

Key words: artificial intelligence, Python, FeedbackFruits, higher mathematics education, digital technologies

JEL Code: A22, I23, C88

Introduction

Mathematics education faces challenges such as maintaining student engagement and connecting abstract mathematical concepts to practical contexts（Zawacki-Richter et al., 2019). Traditional teaching methods can struggle to demonstrate the relevance of mathematics, leading to reduced motivation. Modern technologies, including interactive visualizations, simulations, and programming tools, offer new ways to illustrate mathematical principles and encourage active learning (Perkel, 2018; Lo, 2023）.

At the Prague University of Economics and Business (PUEB), required mathematics courses (e.g. Mathematics for Economists – 4MM101, Calculus – 4MM109, Mathematics for Data Analysis – 4MM110, Mathematics for Informatics – 4MM106/4MM107) cover foundational topics such as functions of one and two variables, linear algebra, limits of functions and sequences, derivatives and their applications, integrals and their applications, and differential equations, see (Klůfa, Pasáčková, 2025) for more details. These topics often challenge students and thus benefit from teaching strategies that make learning more engaging and relevant.

Recent research indicates that generative artificial intelligence, such as large language models, has considerable potential to support both students and instructors in higher education (Kasneci et al., 2023). For example, ChatGPT can provide personalized explanations, generate example problems, and offer immediate feedback on solutions (Lo, 2023). However, these tools should be integrated with clear pedagogical objectives and used as complements to, rather than replacements for, traditional instruction (Kasneci et al., 2023).

Recent studies presented at the International Days of Statistics and Economics conference have highlighted the educational potential of Microsoft Copilot in teaching statistics and solving mathematical problems. The reported findings suggest that generative AI can effectively support students' understanding of quantitative subjects and enrich university teaching (Kaspříková, 2024, Kaspříková, 2025).

The aim of this paper is to present a framework of practical learning activities that integrate generative AI and Python programming into university mathematics courses. We focus on how instructors can incorporate these technologies into existing courses in a meaningful way. We provide examples of using AI tools to explain mathematical procedures, generate problems, and analyze student reasoning, as well as using Python for visualization and verification of mathematical concepts. We also discuss the use of the FeedbackFruits platform for creating interactive learning tasks and assessing student understanding. The goal is to illustrate the potential of these tools in mathematics education and to motivate instructors to adopt them responsibly (Kasneci et al., 2023; Chan, 2023).

1 Didactic objectives and assessment

1.1 Objectives

The primary didactic objective is to deepen students' understanding of key mathematical concepts and to enable them to experiment with functions, equations, and data in real time using digital tools. By combining AI, Python, and active-learning strategies, students can:

- Receive immediate feedback: ChatGPT can explain solution procedures or verify results immediately after a student attempts a problem, while Python can compute exact solutions and generate visualizations（Kasneci et al., 2023).
- Enhance visualization and intuition: Python libraries enable quick plotting of functions, vector representations, and simulations, which support understanding of abstract concepts（Perkel, 2018）.
- Develop independence and creativity: Students can formulate their own queries for AI tools, experiment with code, and explore mathematical hypotheses autonomously.
- Strengthen digital and critical-thinking skills: Students learn basic programming and information verification, and become aware of the limitations and risks of AI-generated content (Chan, 2023; Fangohr, 2004).

1.2 Assessment and Learning Metrics

Instructors can monitor both traditional learning outcomes (tests and assignments) and students' interaction with digital tools. Methods include:

- Pre- and post-tests: Measure improvements in understanding after activity-based instruction.
- Analysis of learning artifacts: Evaluate student-written code or interaction logs to identify misconceptions.
- Reflective assignments: Require students to describe how AI or Python tools contributed to solving a problem.
- Qualitative feedback: Collect student opinions via questionnaires or discussions on the use of AI and programming tools. The goal is not only for students to arrive at correct solutions, but also to document and reflect on their reasoning process.

2 Examples of AI- and Python-Supported Learning Activities

The following section proposes practical classroom activities, each designed with a clear learning objective and supported by AI or Python tools. For each activity, we describe the objective, an example implementation, and expected learning outcomes.

2.1 AI as a Personal Tutor

Objective: Provide personalized explanations of mathematical concepts at the student's level.

Example activity: The student asks ChatGPT a targeted question about a recently covered topic. For example:

"*Explain why the function* $f(x) = \frac{1}{x} + x$ *is increasing on the interval* $(-\infty, -1)$. *Explain it as if teaching a first-year university student*."

The student may continue the conversation with follow-up prompts, such as "*Explain it differently*", "*Use a graph*", or "*Give another example*." This interactive exchange with ChatGPT gives students immediate clarification and multiple perspectives, fostering deeper understanding.

Expected outcomes: Students learn the concept at their own pace, compare different explanations, and improve their ability to articulate mathematical reasoning.

2.2 Personalized Practice Problems

Objective: Provide students with custom practice problems tailored to their skill level.

Example activity: The student uses AI to generate practice exercises. For example:

"*Generate five derivative practice problems, starting with simple examples and gradually increasing the difficulty. Do not provide the solutions*."

The student attempts the problems independently, then asks the AI to check their work. For instance: "*Here is my solution to problem 3. Identify any mistake in my solution without giving the correct answer*." This approach gives students additional practice and instant feedback on errors, guiding further study.

Expected outcomes: Students obtain varied practice problems and learn to self-assess by comparing their answers with AI feedback.

2.3 Verification of Analytical Solutions Using Python

Objective: Use Python to verify students' hand-derived solutions, reinforcing mathematical understanding.

Example activity: After solving a problem analytically, the student checks the result with Python. For example, to verify a derivative:

```
from sympy import symbols, diff
x = symbols('x')
f = x**3 - 2*x**2 + 5*x - 7
diff(f, x)
```

Python's output confirms or corrects the student's result. This shows students how symbolic computation can validate their work.

Expected outcomes: Students confirm their solutions, understand the role of software in verifying mathematics, and learn to interpret computer-algebra output.

2.4 Visualization of Mathematical Concepts

Objective: Enhance comprehension of abstract concepts by plotting and visualizing.

Example activity: After computing a definite integral by hand, the student asks the AI what is the meaning of the value and then uses Python to visualize the area. For instance:

```
import numpy as np
import matplotlib.pyplot as plt
x = np.linspace(0, 2, 100)
y = x**2 + 1
plt.plot(x, y)
plt.fill_between(x, y, color='lightblue')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Area under curve of y=x^2+1 from x=0 to 2')
plt.show()
```

This plot visually represents the integral $\int_0^2 x^2 + 1 \, dx$.

Expected outcomes: Students relate symbolic calculations to graphical intuition, understanding how the integral corresponds to the shaded area.

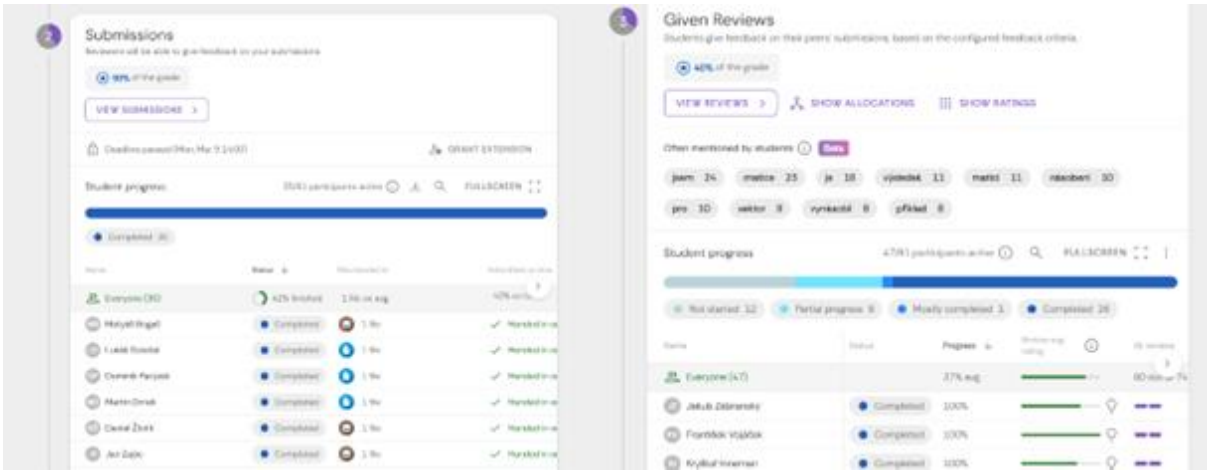
2.5 FeedbackFruits – Peer Review

Objective: Develop critical thinking and communication by having students evaluate each other's work.

Example activity: Using the FeedbackFruits peer-review tool, each student submits a solution to a given problem. The system then assigns each student two anonymous solutions from peers to review. Students assess correctness, point out errors, and provide constructive feedback. Before submitting their reviews, students may ask an AI assistant for suggestions on how to give helpful feedback.

For example, students are asked only to design a practical problem involving the use of matrix calculations (e.g. in economics, engineering, or another real-world application). They may draw inspiration from online resources such as Google or AI tools. Afterwards, each student is assigned two problems created by their classmates. They solve these problems and evaluate their practicality and level of difficulty. Finally, they read the feedback and comments provided by their classmates on their own problem.

Fig. 1: Example of the FeedbackFruits peer review activity



Source: The moodle course of 4MM106.

Expected outcomes: Students improve their own understanding by critiquing others, learn to communicate mathematical ideas clearly, and develop evaluation skills.

2.6 FeedbackFruits – Interactive Video

Objective: Increase engagement during video-based lessons and ensure comprehension.

Example activity: Create an interactive instructional video on limits or derivatives, embedding questions at key points. For instance, after explaining a concept, the video pauses and asks a quiz question that students must answer to continue. The FeedbackFruits platform records responses.

Expected outcomes*:* Students actively process content while watching videos, and instructors receive real-time formative data on student understanding.

2.7 FeedbackFruits – Comprehension

Objective: Gather student feedback on learning and identify misconceptions after a lesson.

Example activity: After a class session, use FeedbackFruits Comprehension activity: each student answers questions such as "*Which concept is still unclear*?" or "*What example helped you the most?*" Responses are collected anonymously. The instructor reviews this feedback before the next lesson.

Expected outcomes: Early detection of topics that require reinforcement, enabling adaptive teaching.

2.8 AI-Assisted Error Analysis

Objective: Encourage students to identify and correct their own mistakes using AI guidance.

Example activity: Students use ChatGPT to find errors in their own solutions. For example:

"*Here is my solution to the equation. Do not solve it completely; only identify the first error in my reasoning and explain why it is incorrect.*"

The AI points out the mistake without giving away the full solution.

Expected outcomes: Students learn to critically evaluate their work and understand errors, promoting metacognitive skills.

2.9 AI-Generated Economics Problems

Objective: Create relevant application problems to connect mathematics with economics.

Example activity*:* Students ask ChatGPT to generate word problems involving economic contexts. For instance:

"*Create a business optimization problem that involves finding the maximum profit, which requires using derivatives to solve*."

Students then solve the problem and verify with Python.

Expected outcomes: Students apply mathematical methods to practical scenarios, reinforcing both their mathematical and economic understanding.

2.10 Combining AI and Python Tools

Objective: Integrate multiple tools in a single, coherent learning task.

Example activity: Students solve a problem manually, verify it with Python, and use AI for conceptual insight. For example, after finding a derivative of a cost function and verifying it in Python, the student asks:

"*What does this derivative represent in the context of the cost function?*"

ChatGPT provides an economic interpretation (e.g. marginal cost analysis).

Expected outcomes: Students reinforce their solution through multiple representations (symbolic, computational, verbal) and gain a deeper understanding of the meaning behind mathematical results.

3 Implementation in Mathematics Courses

The integration of interactive AI/Python activities can be introduced gradually across mathematics courses. In courses like Mathematics for Economists (4MM101) and Calculus (4MM109), some of these activities are already included in Moodle-based materials as graded homework. They help students practice course content and deepen their understanding.

For larger courses such as Mathematics for Informatics (4MM107) and Mathematics for Data Analysis (4MM110), we plan to create dedicated modules on mathematics with AI and Python. For example, in 4MM107, students will spend part of lab sessions working on mathematical problems using Python. Homework assignments will include tasks that emphasize conceptual understanding and the use of AI tools.

Practical Implementation

- Introduction to tools: Students receive a brief introduction to Python and AI tools, learning basic commands and how to write effective prompts.
- Collaborative work: Activities are done in pairs or small groups (one student writes code or a prompt, another evaluates the AI's output). This encourages discussion and reflection. These tasks can be done in class or as homework.
- Live demonstrations: The instructor may demonstrate an example live (running Python code or a ChatGPT prompt) and then guide students in interpreting and critiquing the results together.

4 Ethical and Technical Limitations

4.1 Ethical Considerations

The use of AI in education raises concerns such as the risk of students copying AI-generated solutions without understanding them and the reliability of AI-provided information（Kasneci et al., 2023; Barrett & Pack, 2023). Therefore, it is important to emphasize that AI is not a "teacher-in-a-box" but a supportive tool. Students should be guided to verify AI outputs (for example, by comparing them with Python results or analytical solutions) and to use them critically (Kasneci et al., 2023). It is also valuable to discuss ethical aspects such as academic integrity (e.g. avoiding plagiarism of AI-generated content), copyright, and the importance of critical thinking when interpreting AI outputs.

4.2 Technical Limitations

Generative AI tools also have limitations. Their responses may be incomplete or incorrect, and they might omit valid solution methods (Kasneci et al., 2023). They sometimes produce plausible but false information (so-called "hallucinations"). For advanced symbolic mathematics, dedicated software or Python (SymPy) is often more reliable. Additionally, use of AI requires internet access and suitable computer infrastructure, which can be a logistical challenge. Finally, many AI platforms collect user data, so instructors should verify that these tools comply with privacy regulations (e.g. GDPR) and institutional policies.

Conclusion

The integration of artificial intelligence and Python into mathematics education at PUEB offers innovative opportunities to make instruction more engaging and to personalize support for students. The proposed activities illustrate concrete ways to combine traditional content with modern tools. While their implementation requires initial effort and attention to technical and ethical constraints, such approaches have the potential to significantly increase student engagement and deepen conceptual understanding (Kasneci et al., 2023; Chan, 2023).

The next step is a pilot introduction of these methods in selected courses, accompanied by collection of feedback (e.g. through test assignments and surveys). Future research could analyze the impact of these methods on students' performance, digital competences, or attitudes toward mathematics. Ultimately, the goal is for modern technologies to provide students with meaningful, comprehensible support in learning mathematics.

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